Nonparametric Bayes and Exchangeability

Tamara Broderick
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Electrical Engineering & Computer Science
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http://www.tamarabroderick.com/tutorials.html

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"goal of presenting the basic techniques, definitions and goals in several of the communities"

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Nonparametric Bayes

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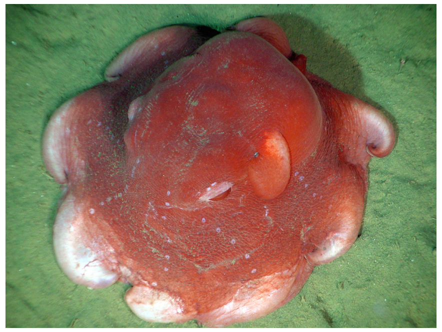

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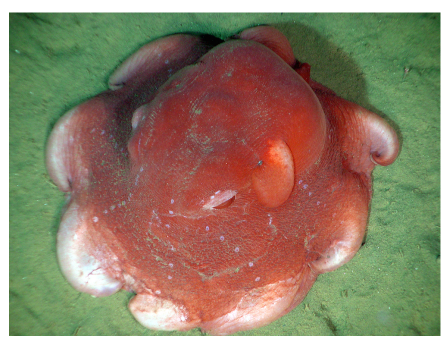

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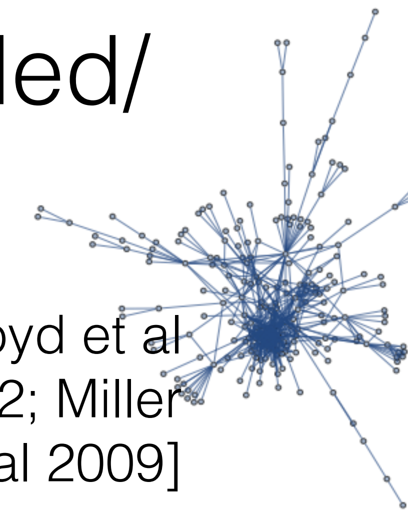

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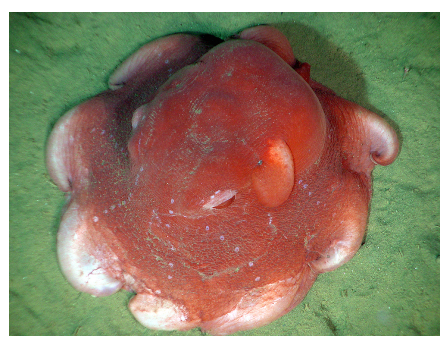

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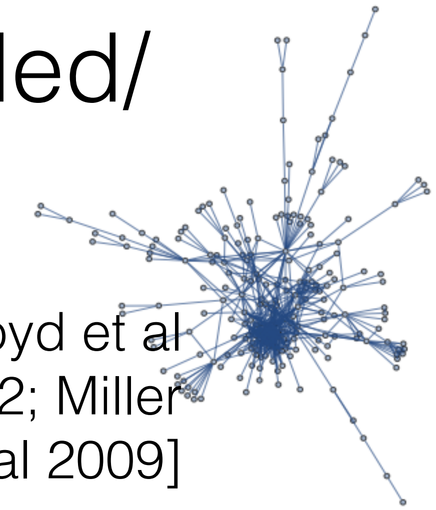

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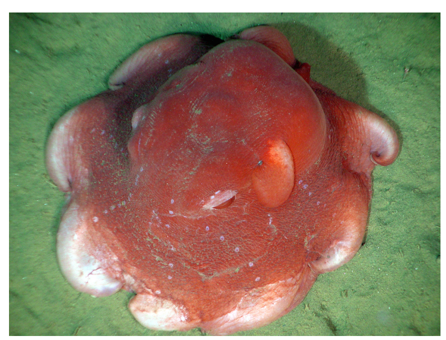

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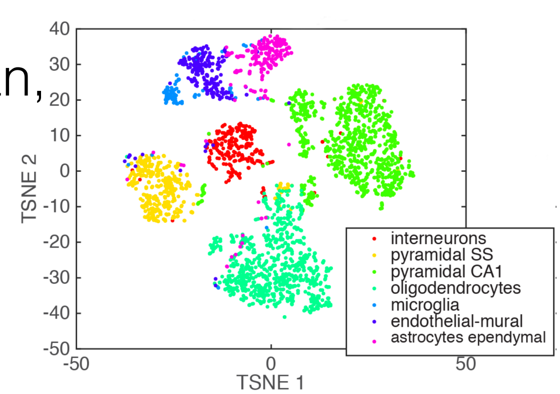

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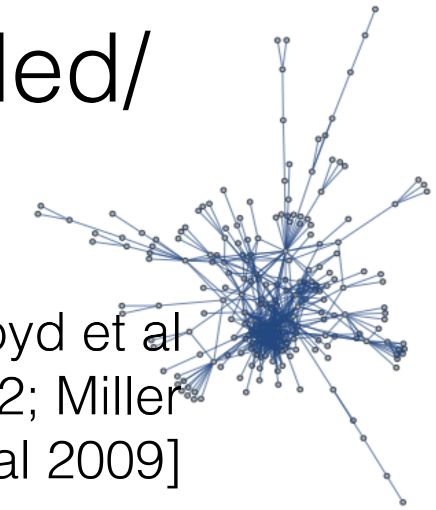

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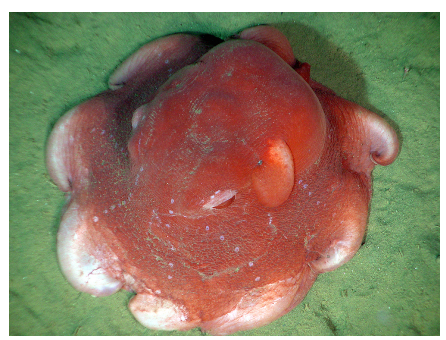

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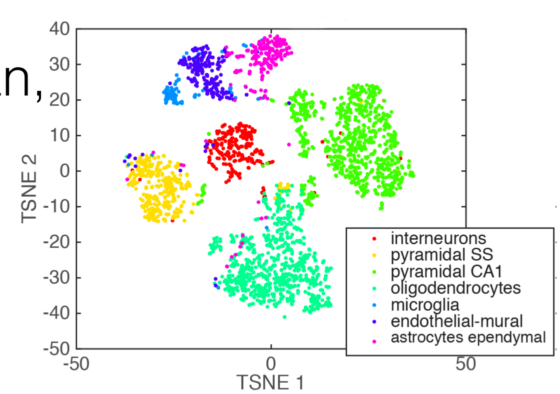

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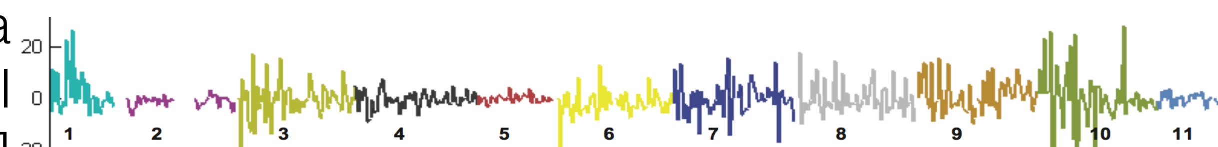

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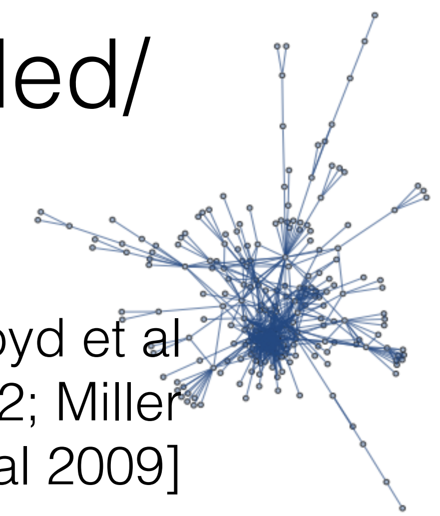

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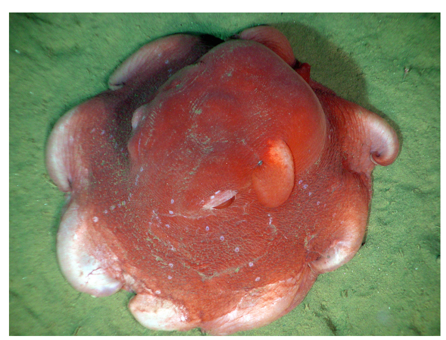

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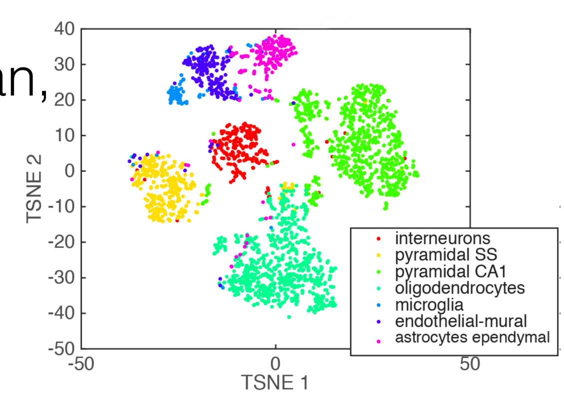

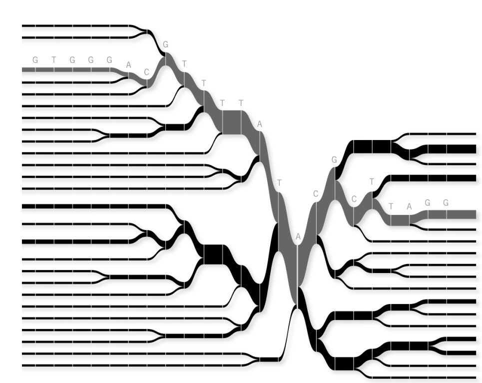

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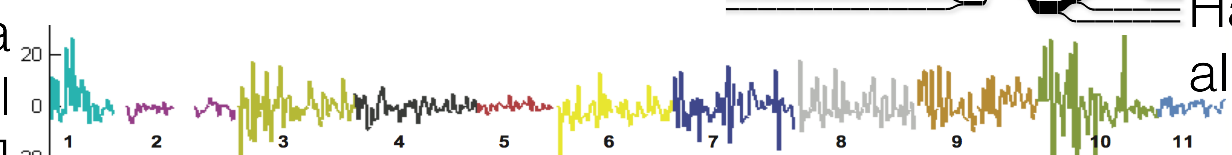

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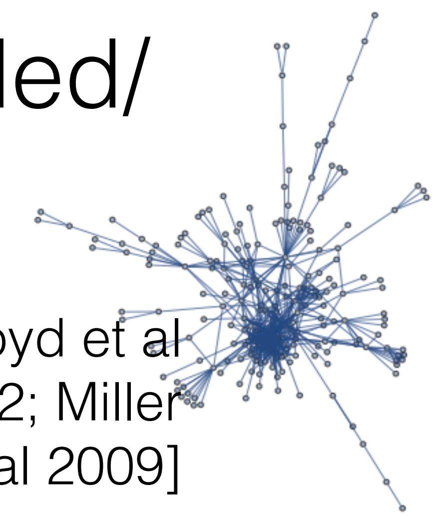

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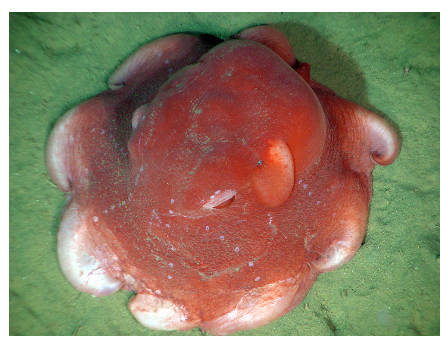

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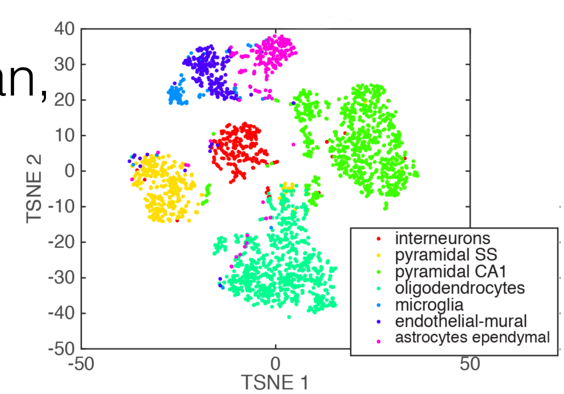

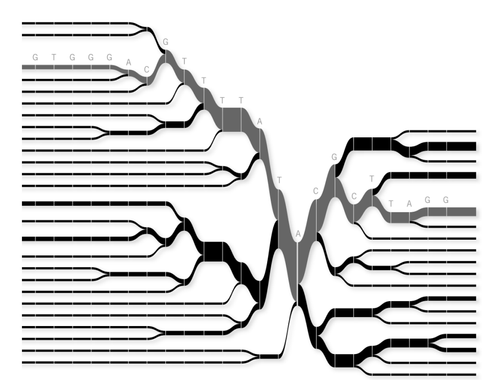

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[ESO/ L. Calçada/ M. Kornmesser 2017]

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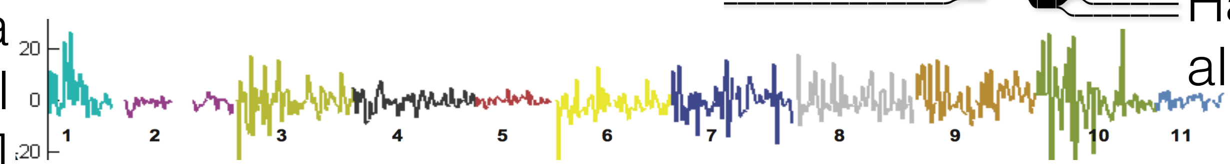

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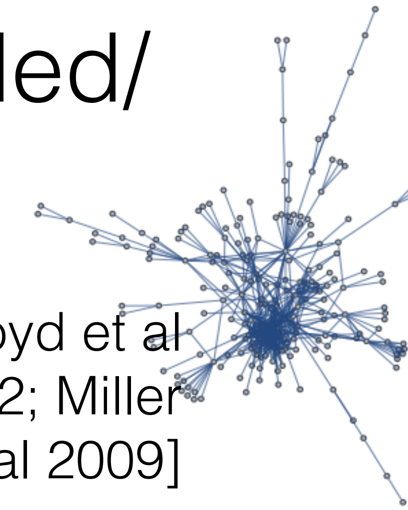

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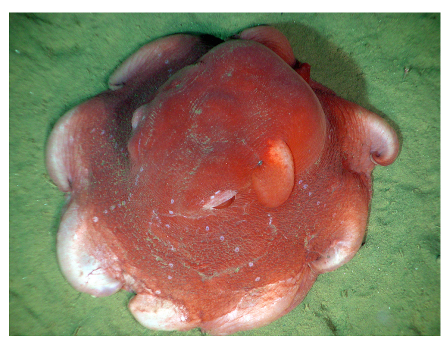

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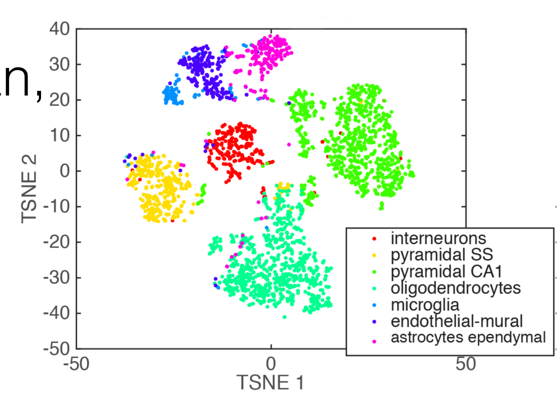

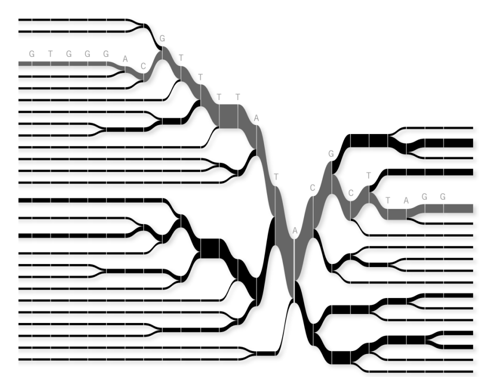

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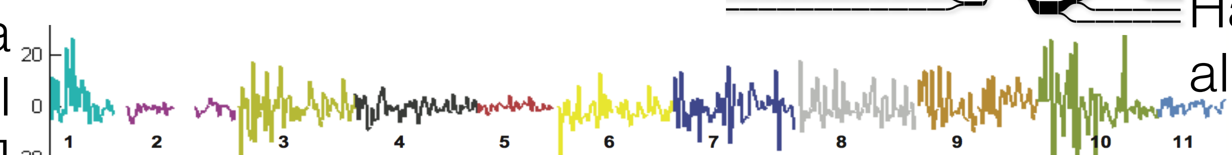

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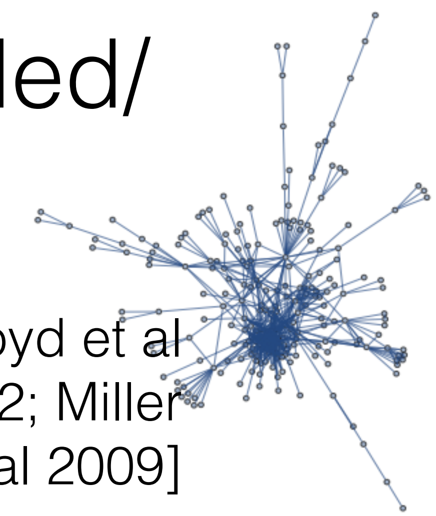

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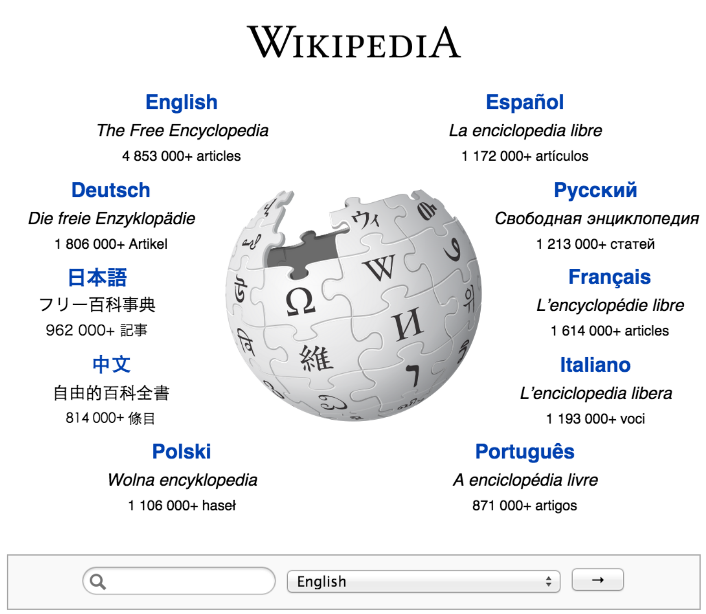

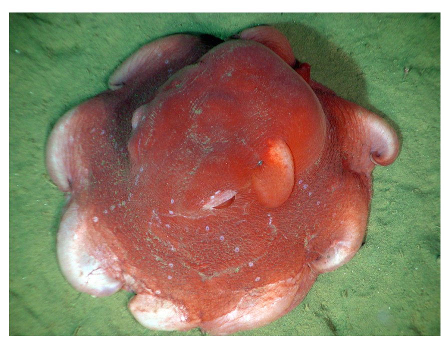

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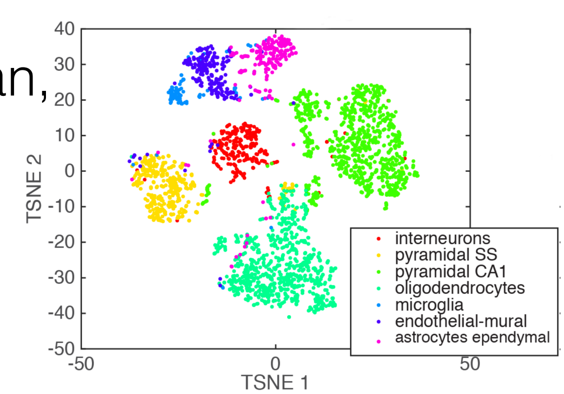

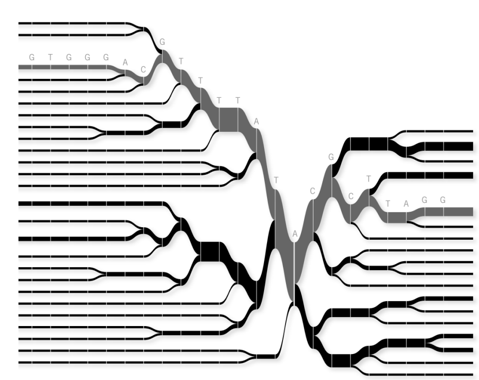

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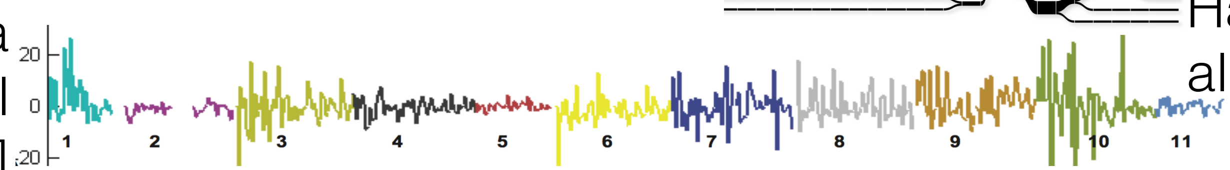

[Cassidy et al 2015]

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  - "Nonparametric Bayesian" priors

[Hewitt, Savage 1955; Aldous 1983]

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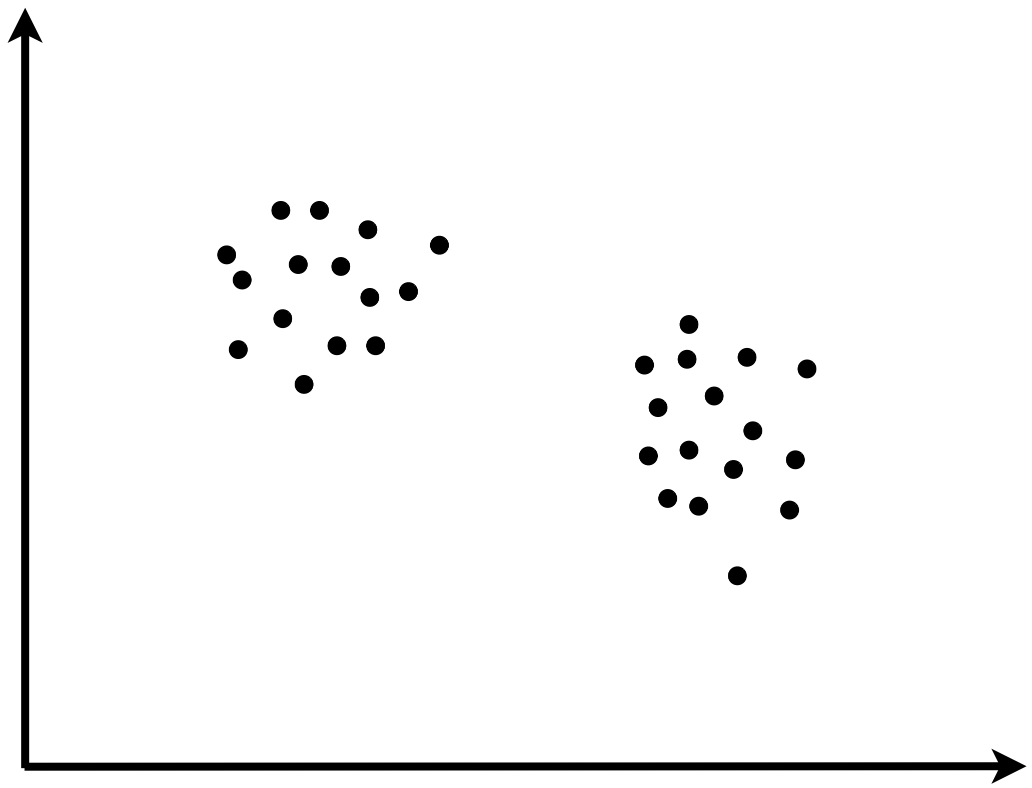
  - How does thinking about exchangeability help us use NPBayes in practice?

Generative model



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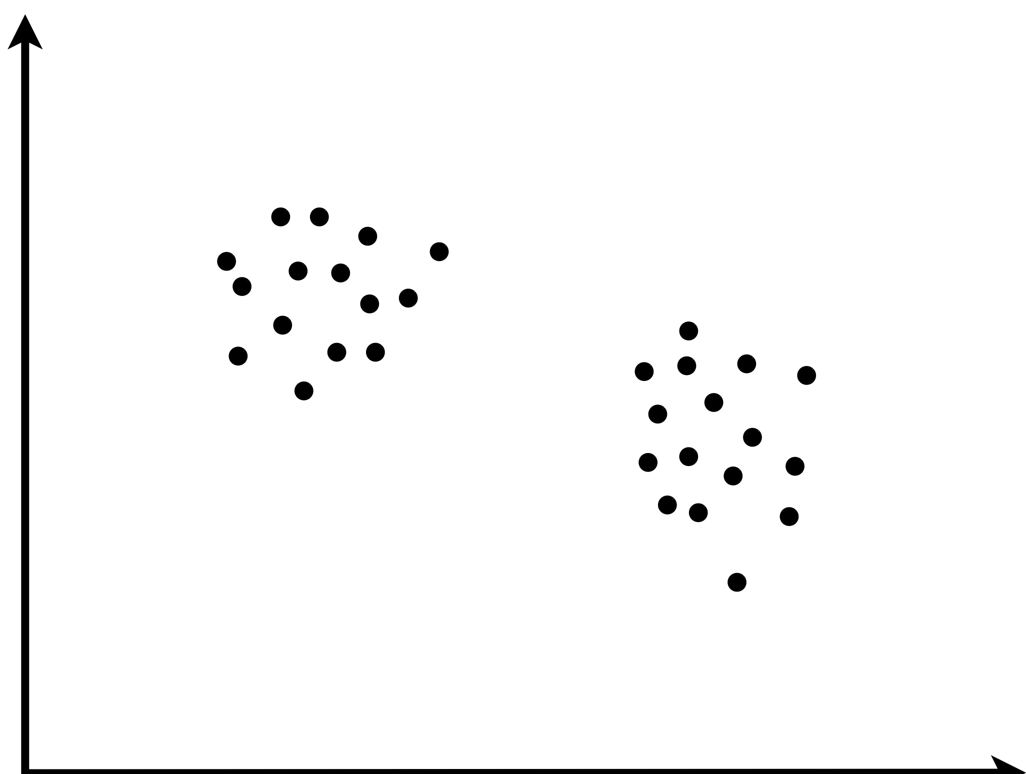

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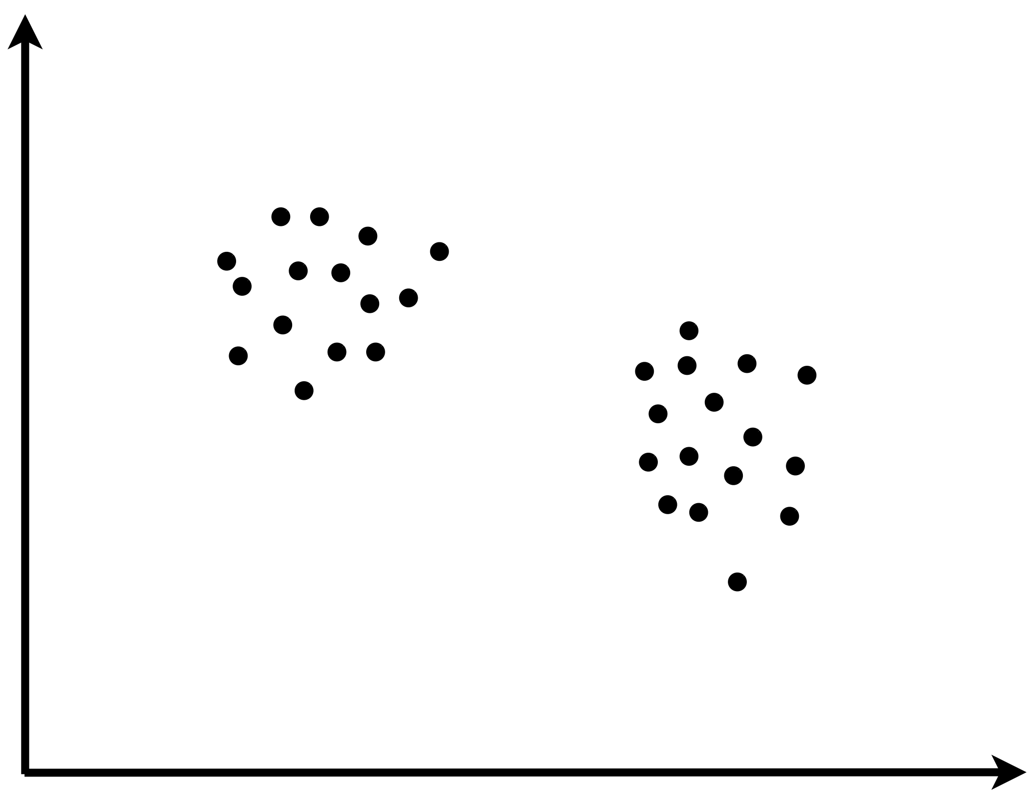

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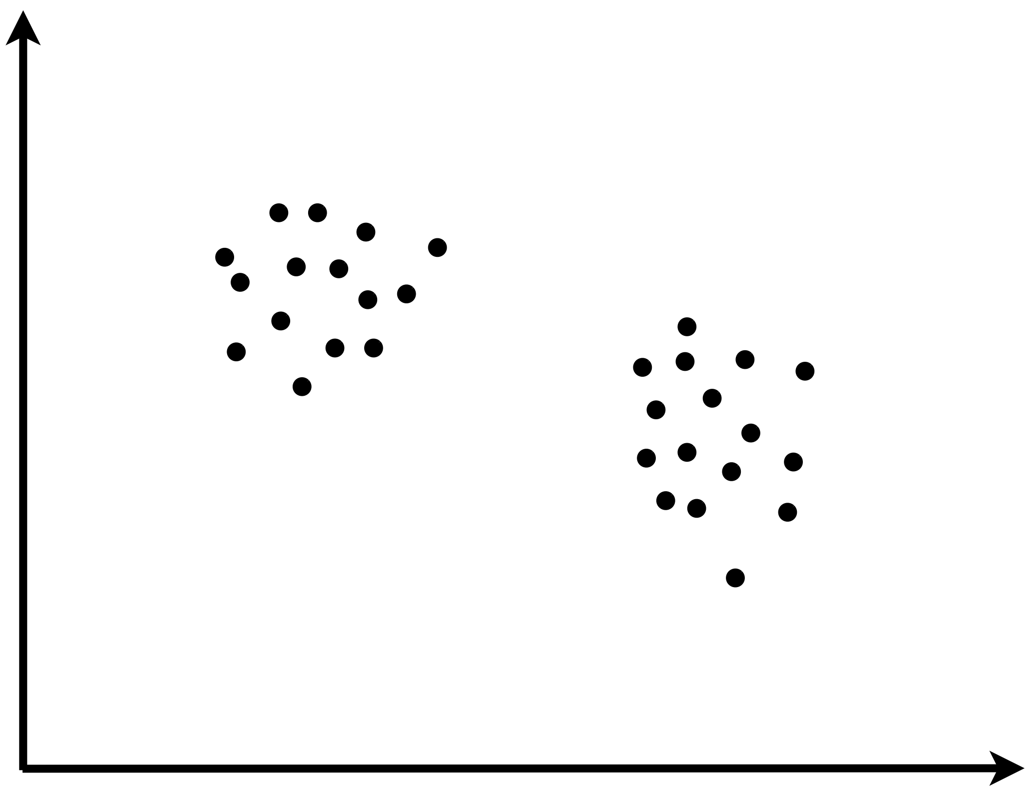

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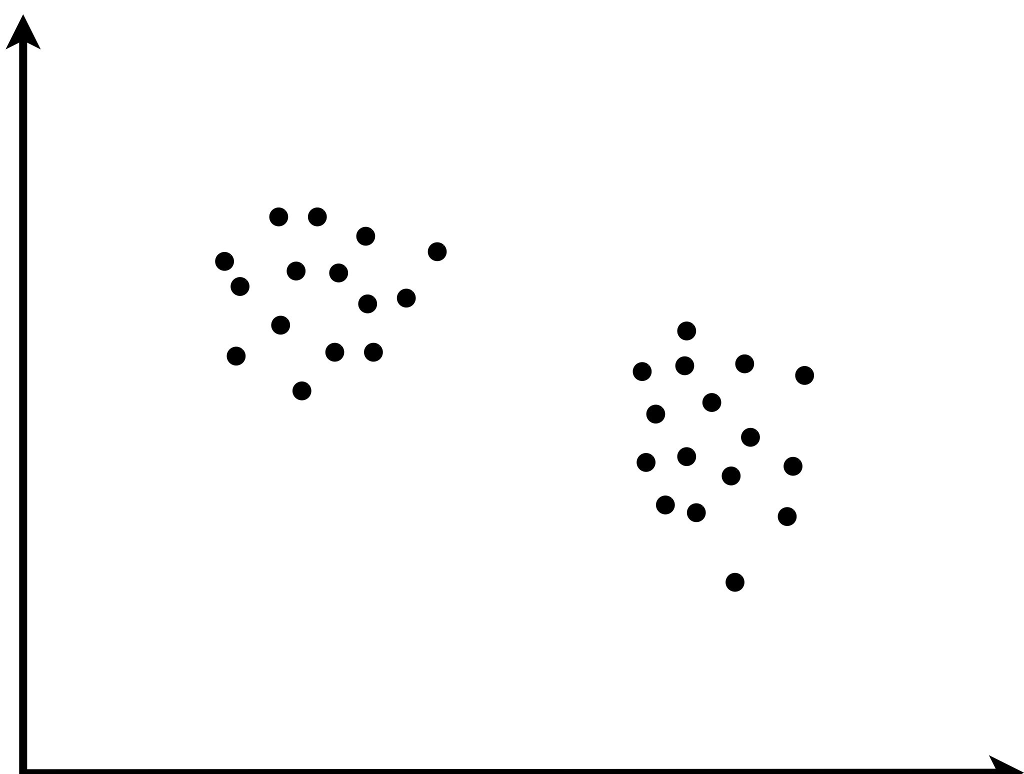

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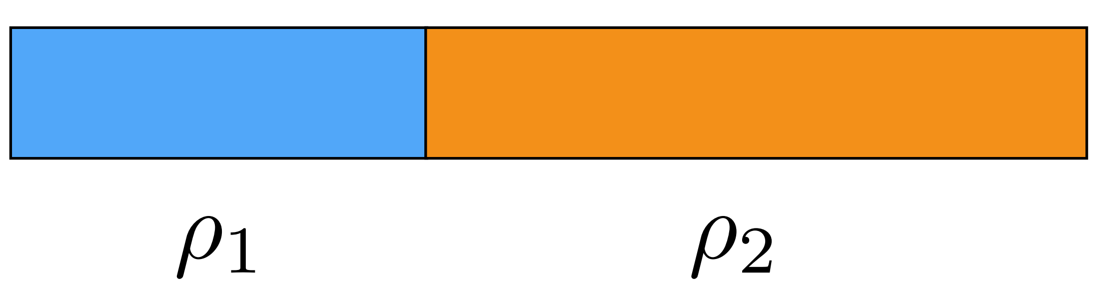

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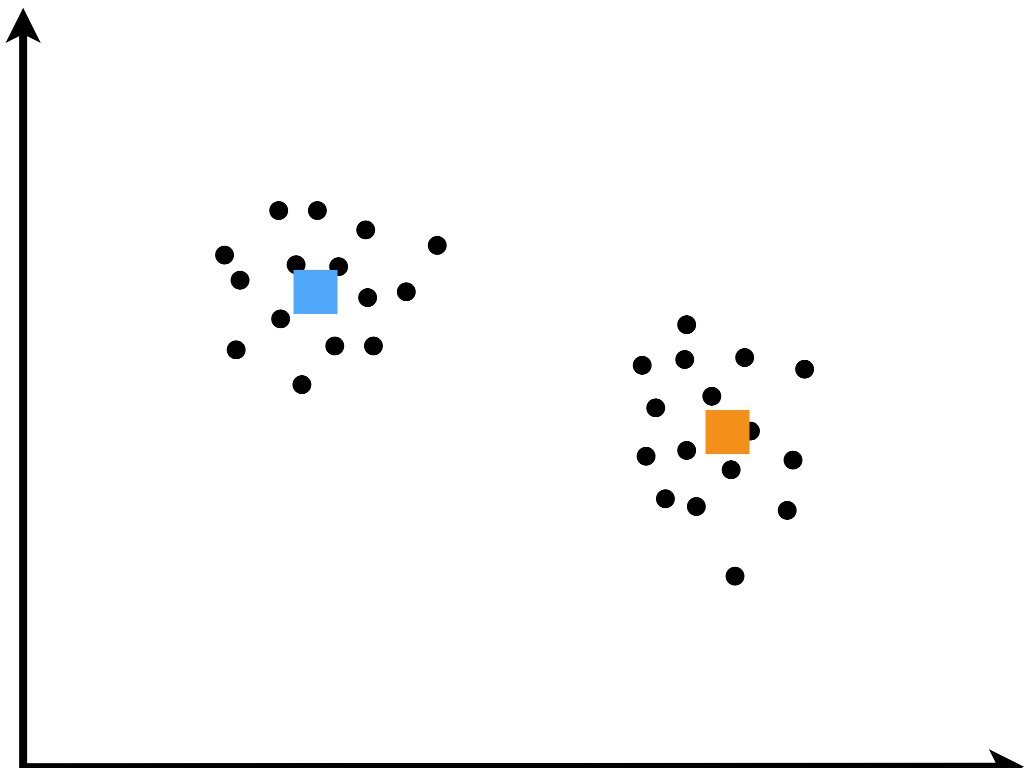

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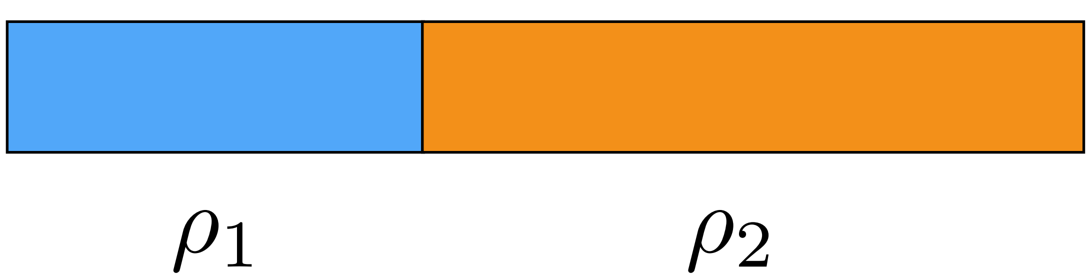
$$x_n \overset{indep}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma)$$



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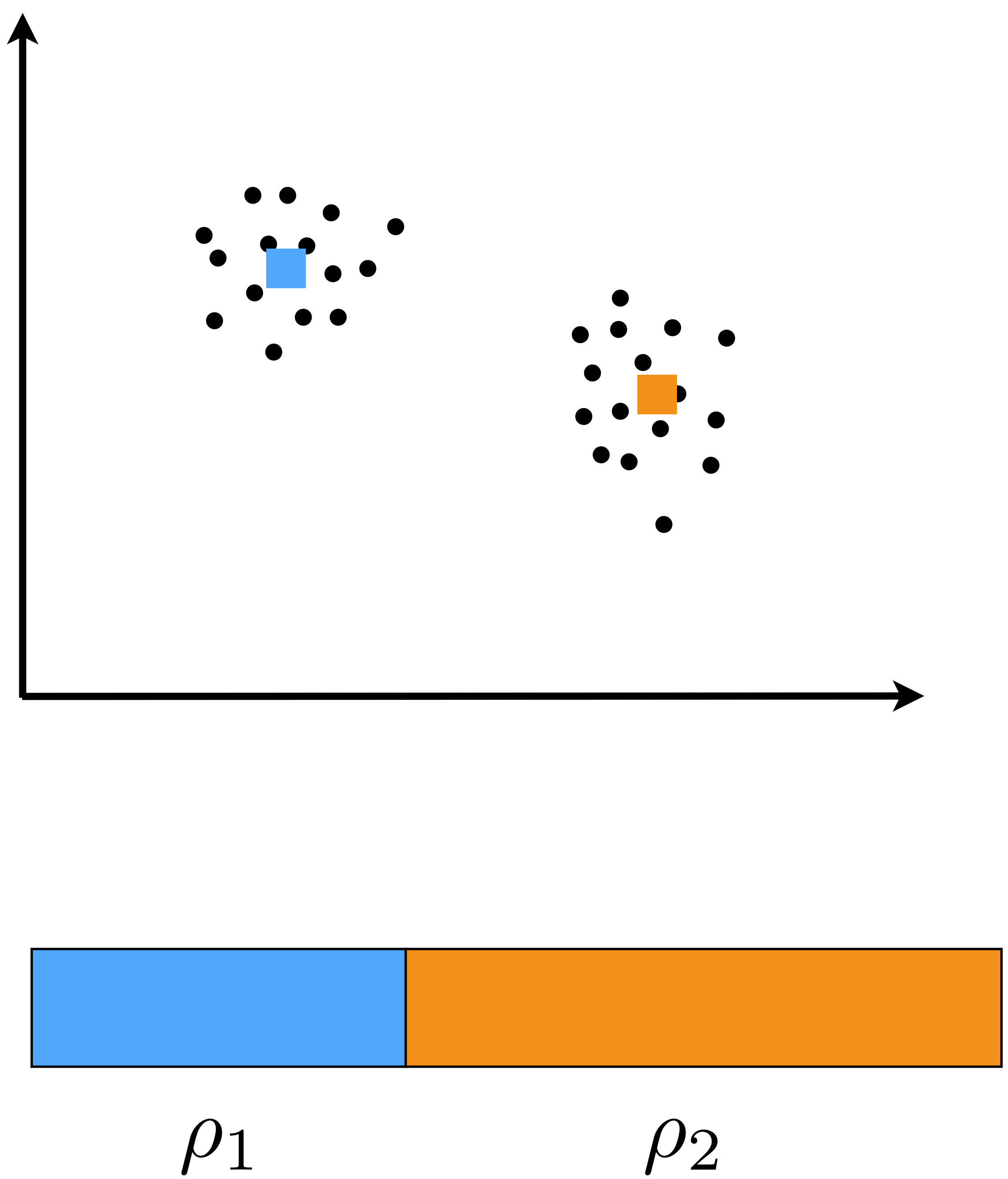

- Finite Gaussian mixture model (*K*=2 clusters)

$$z_n \overset{iid}{\sim} \text{Categorical}(\rho_1, \rho_2)$$
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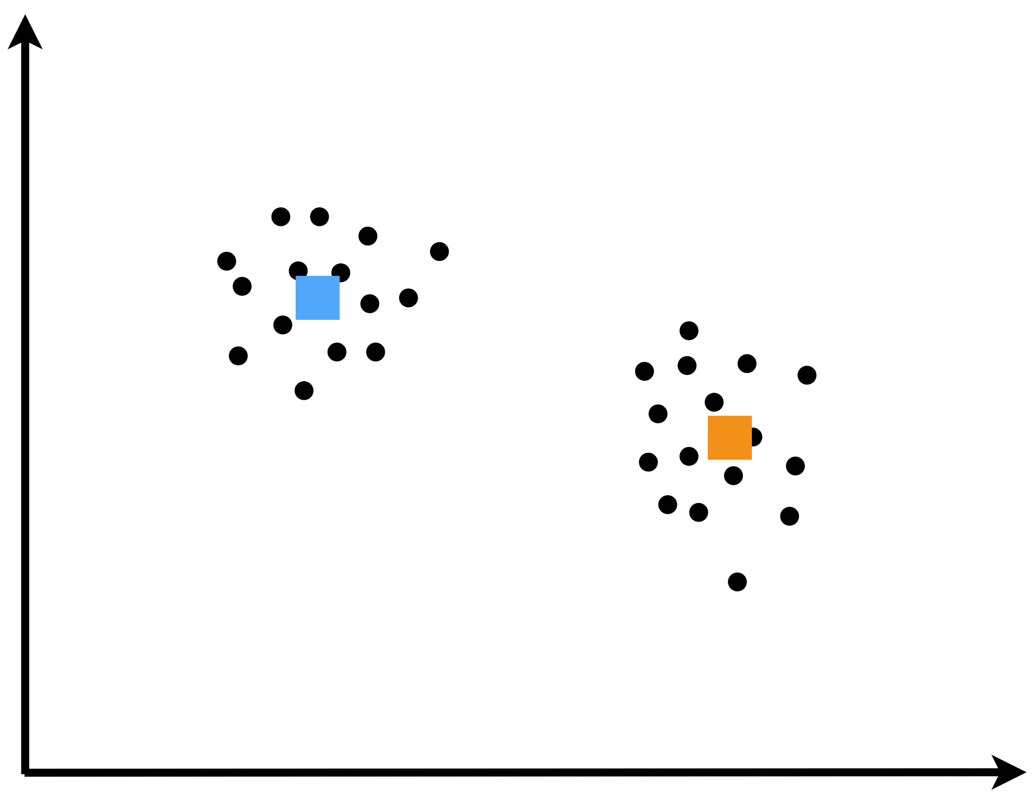

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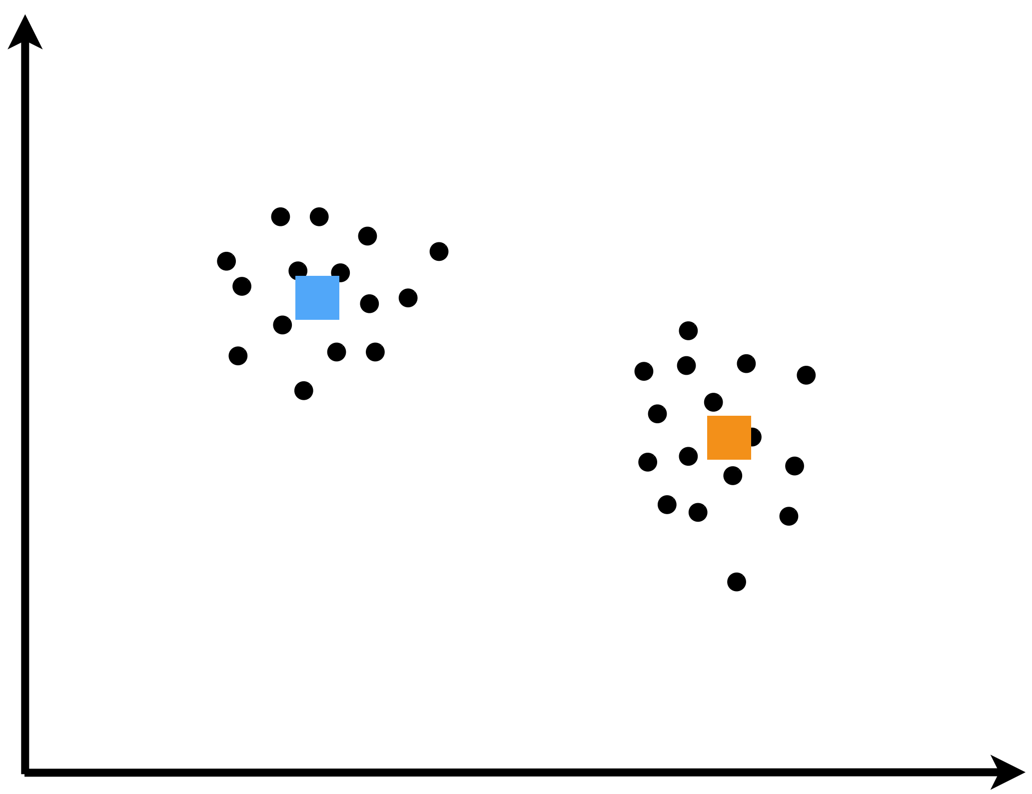

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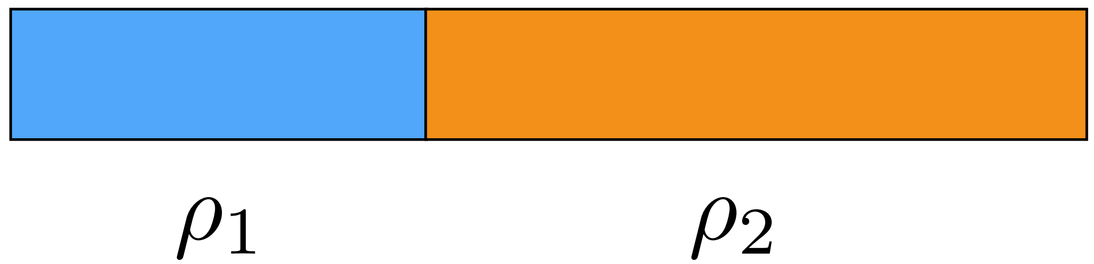

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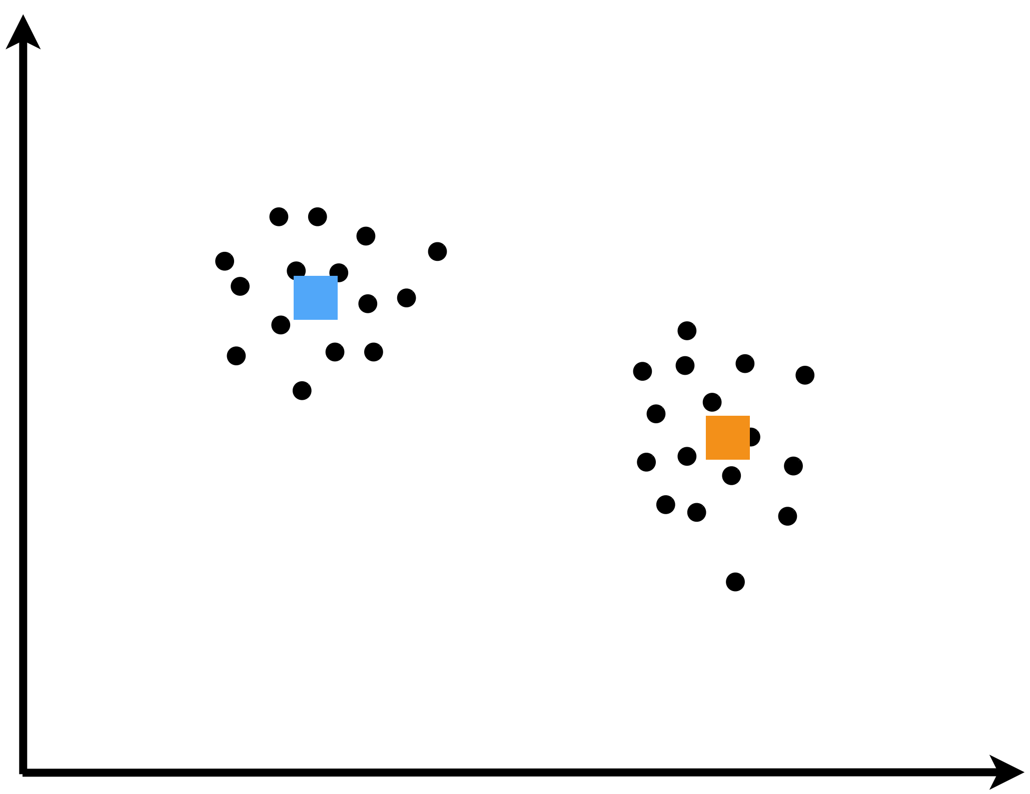

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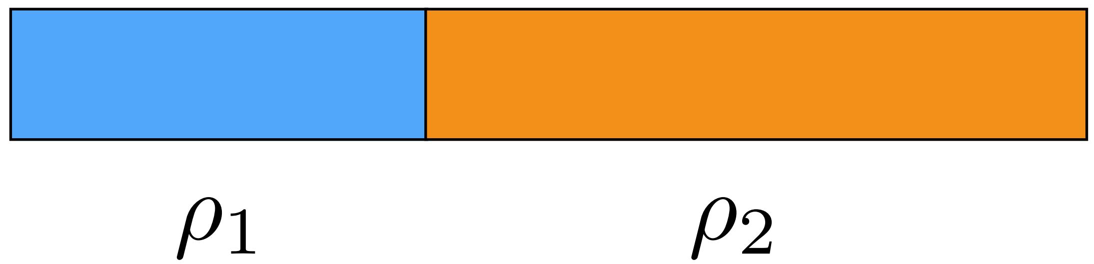

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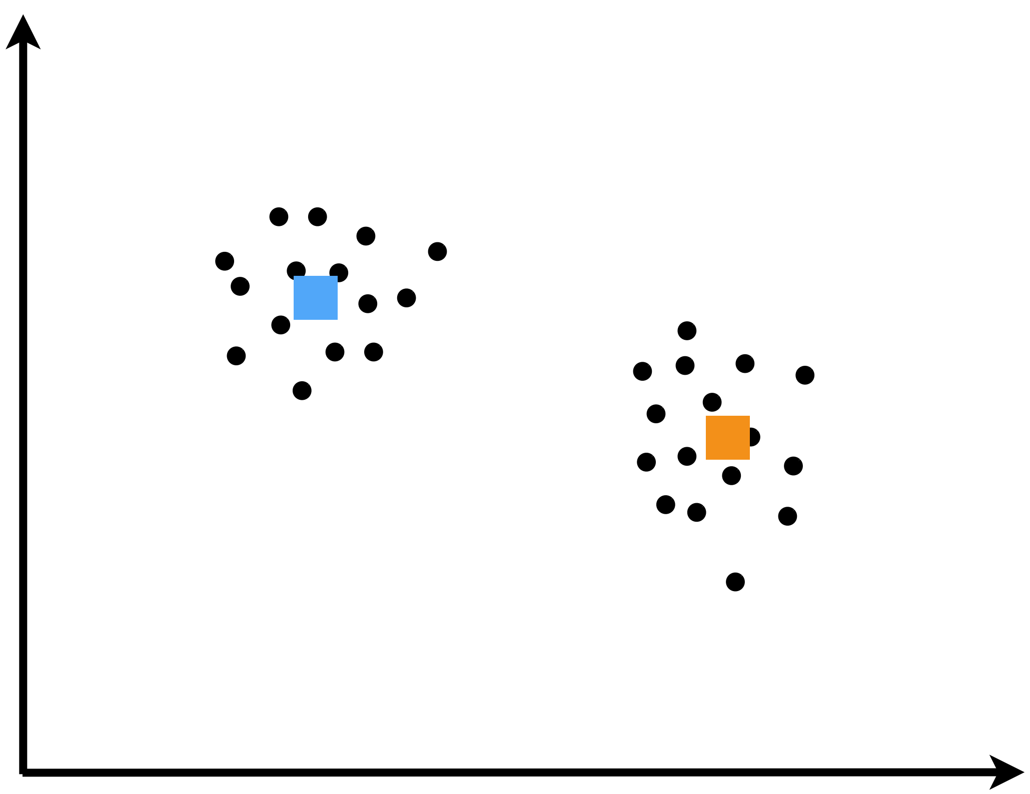

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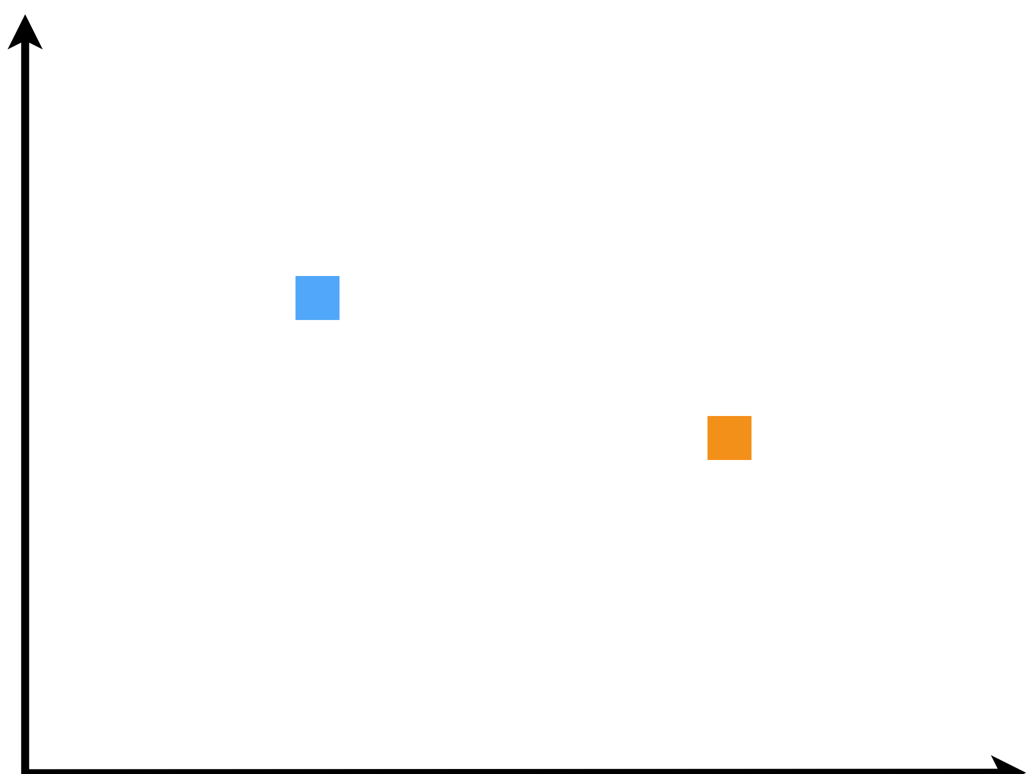

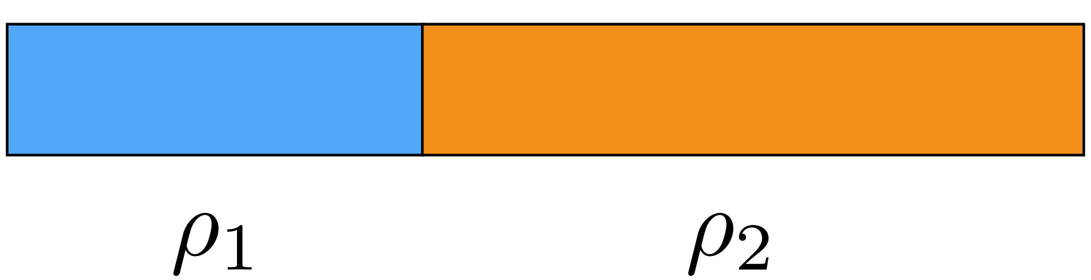

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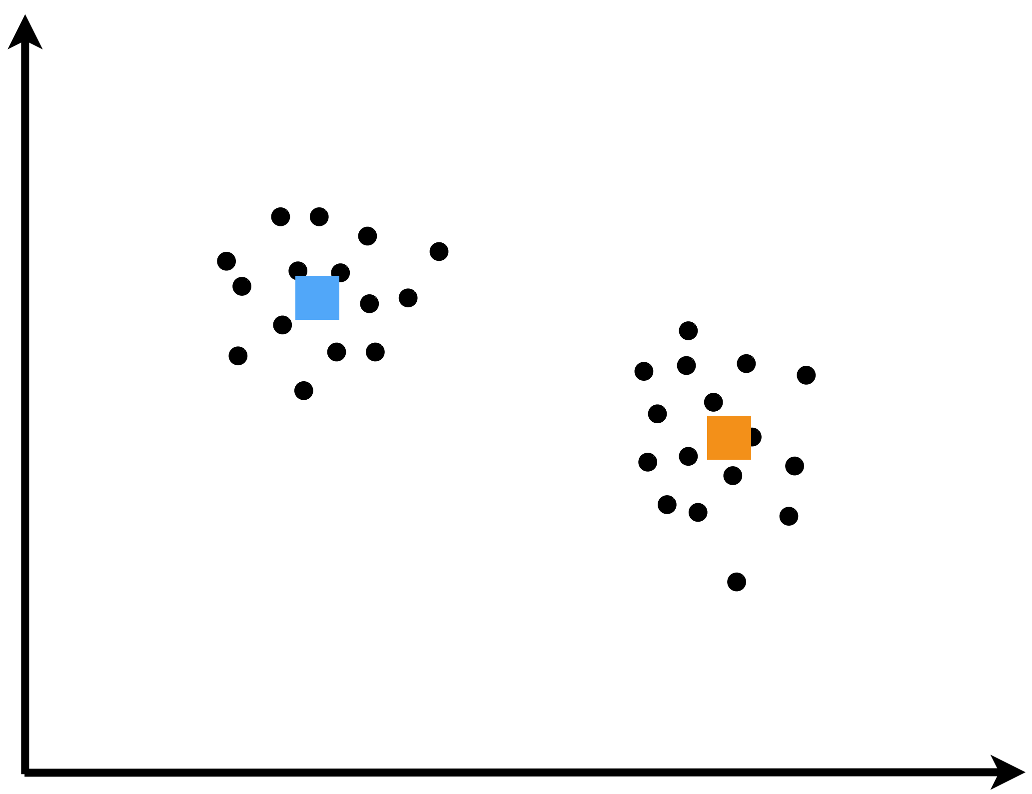

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$\rho_1 \in (0,1)$

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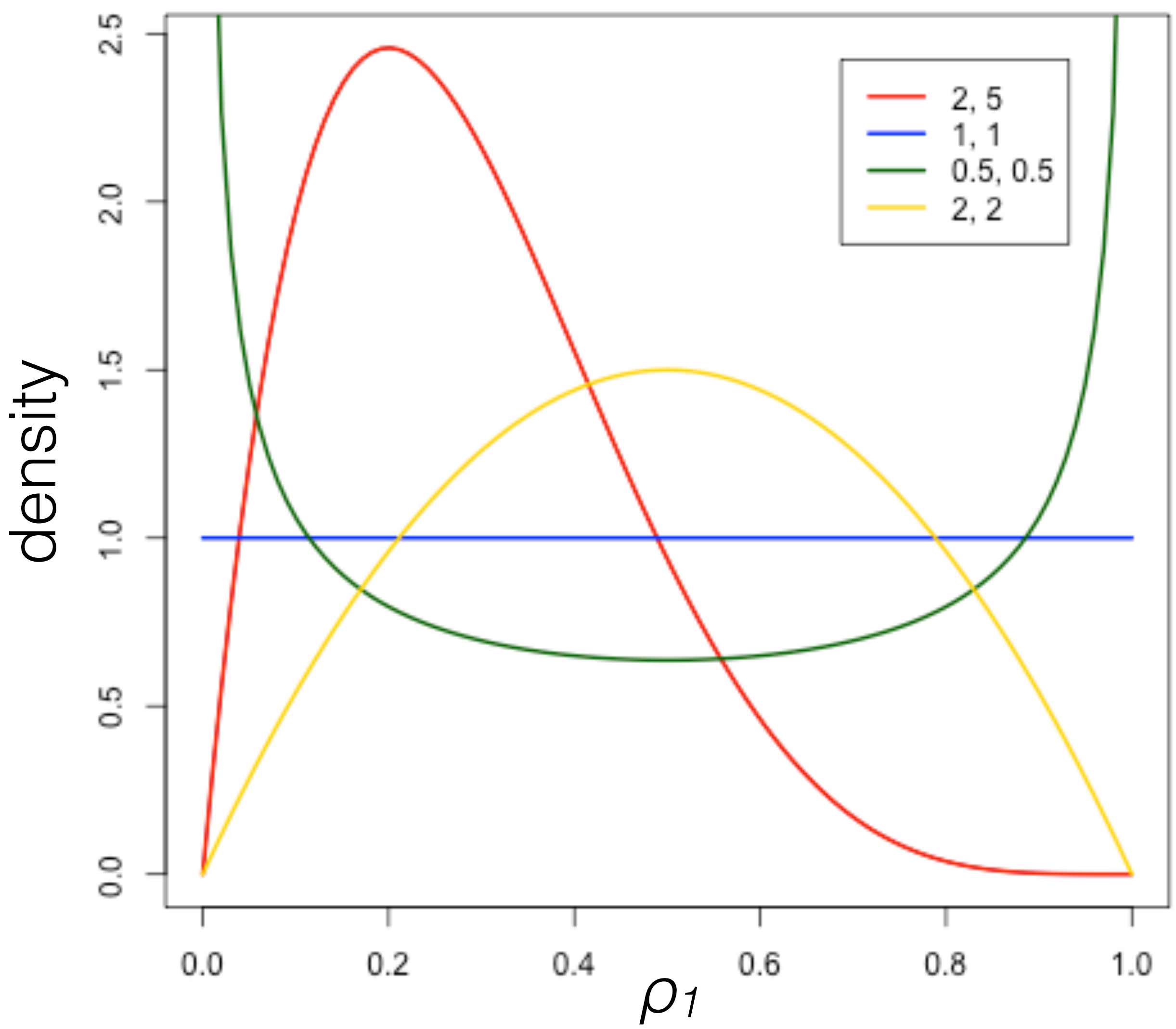

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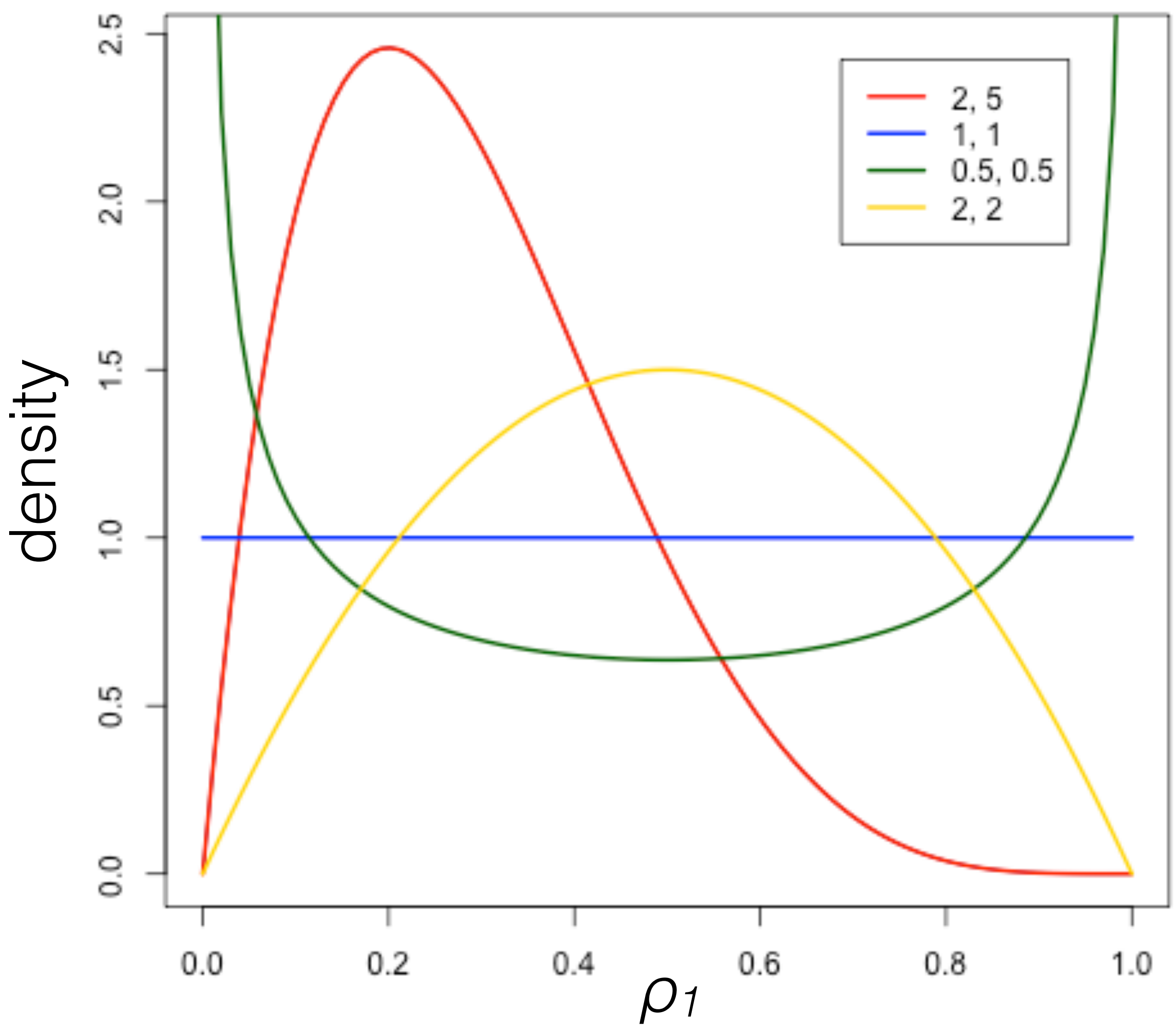

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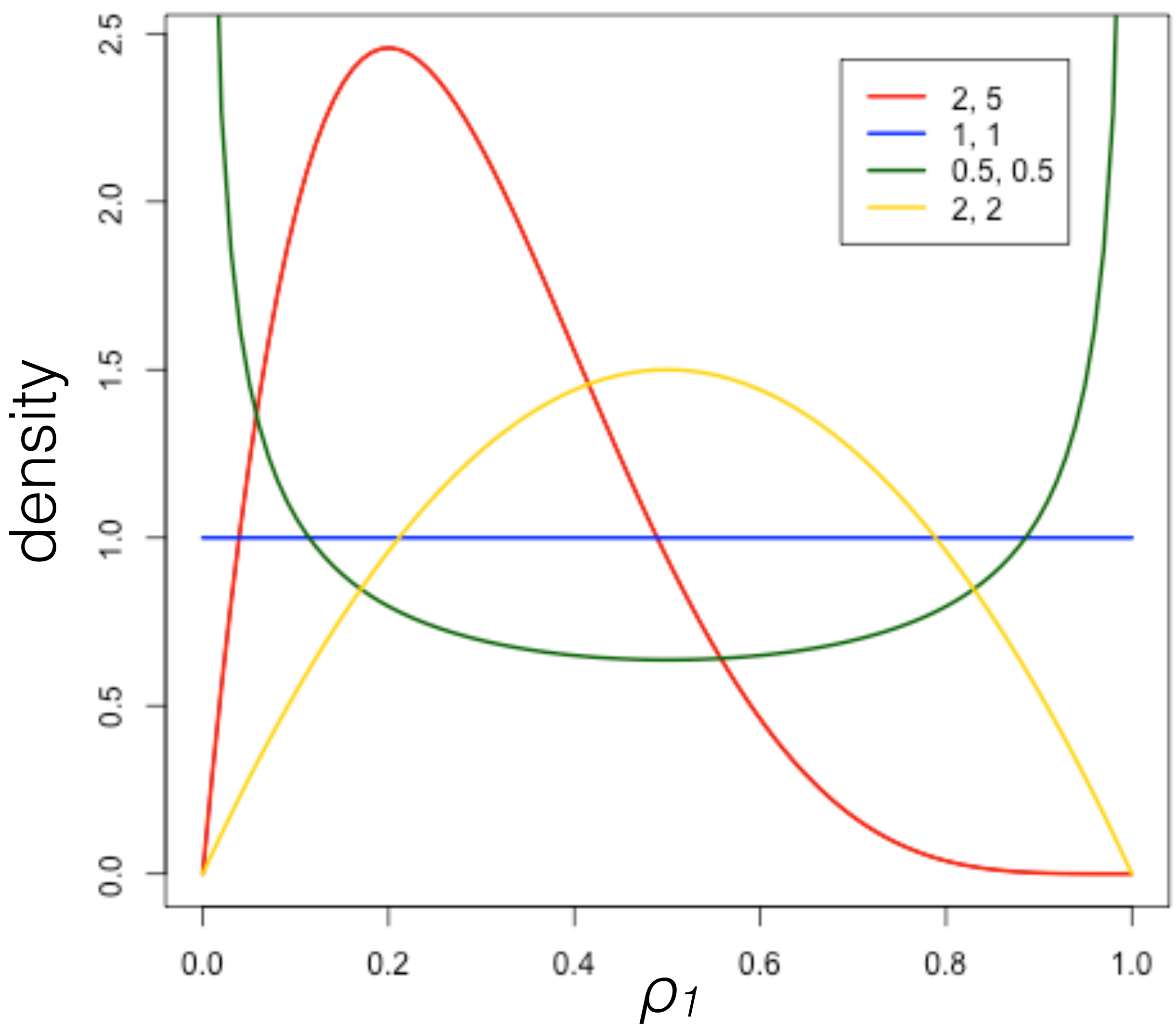

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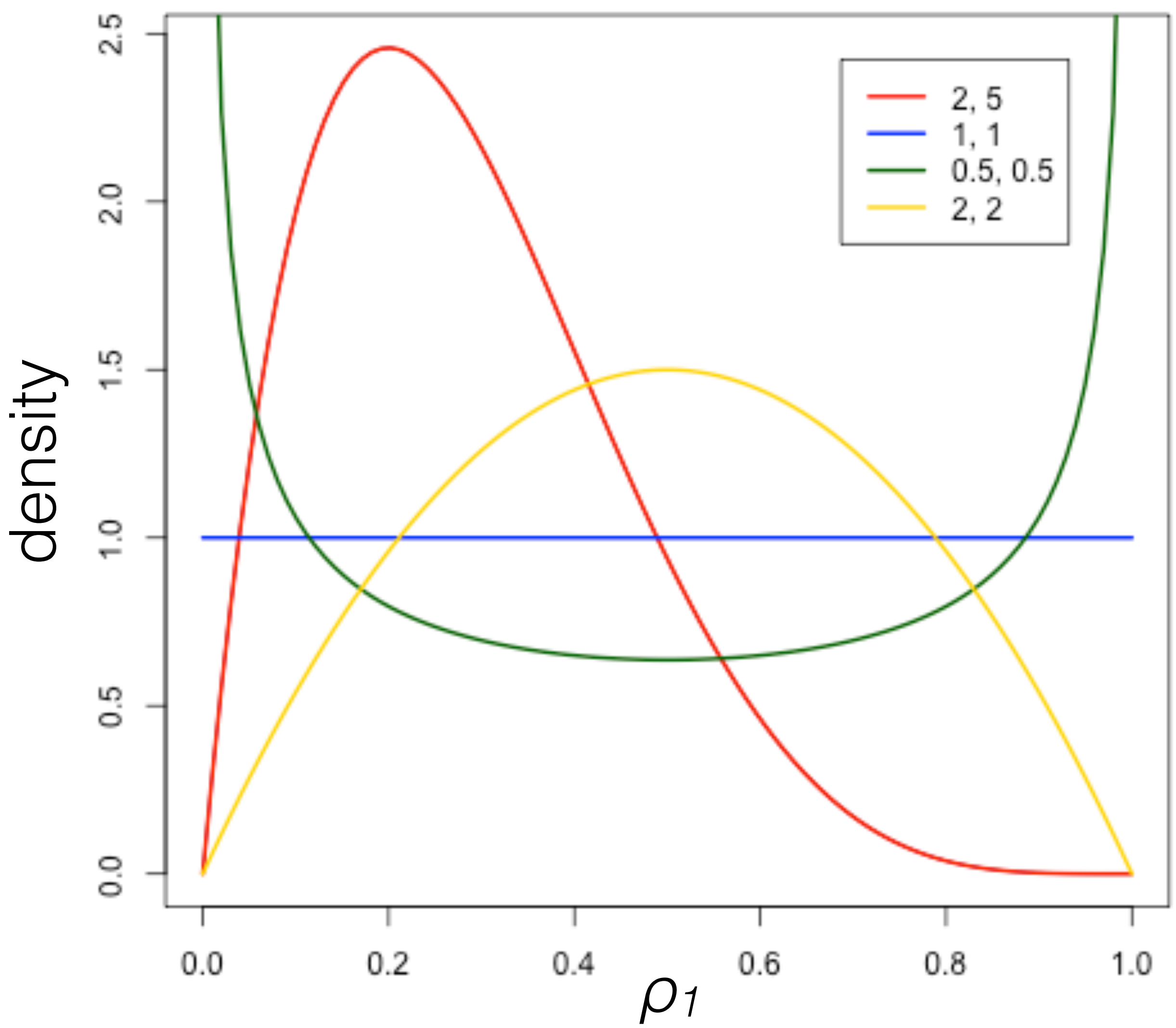

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[demo]

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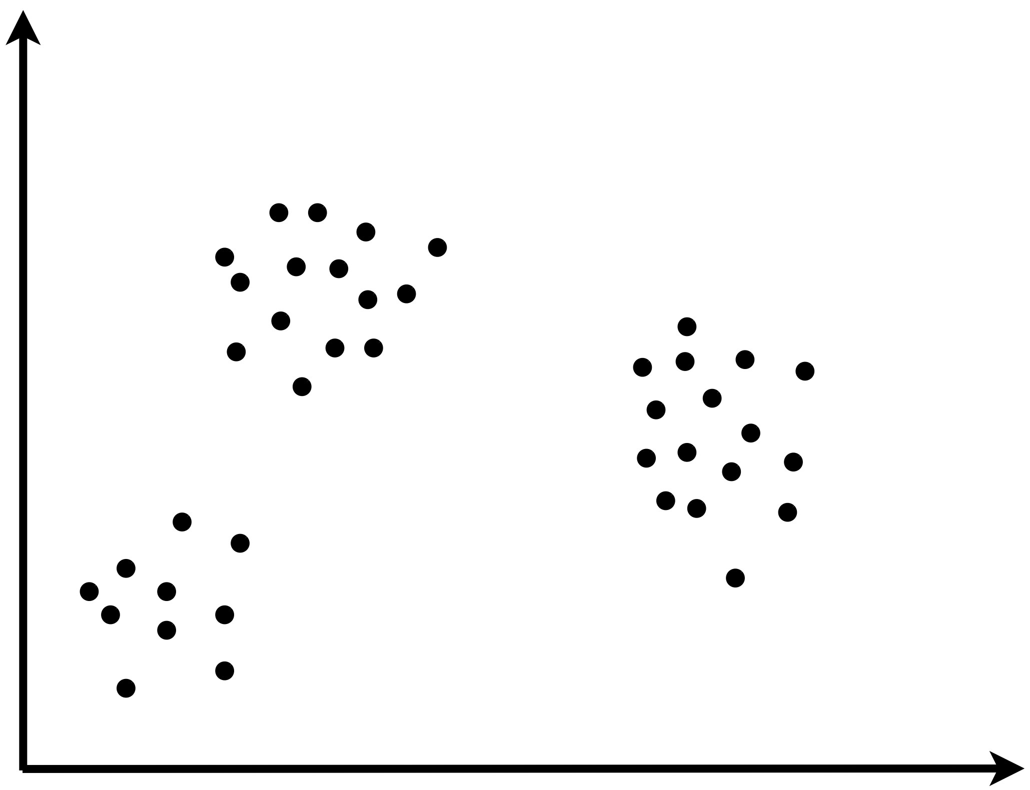

- Finite Gaussian mixture model (K clusters)

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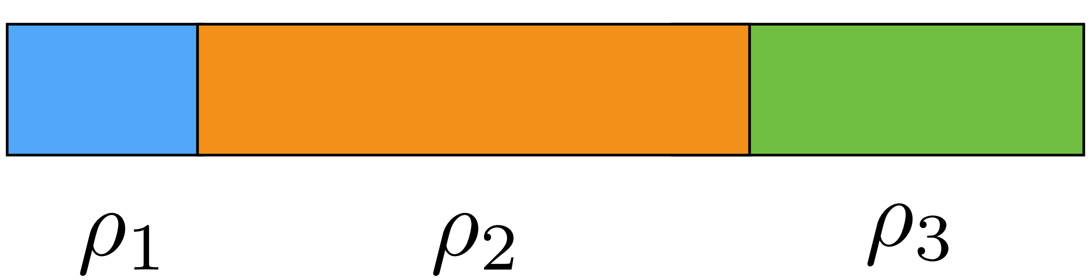

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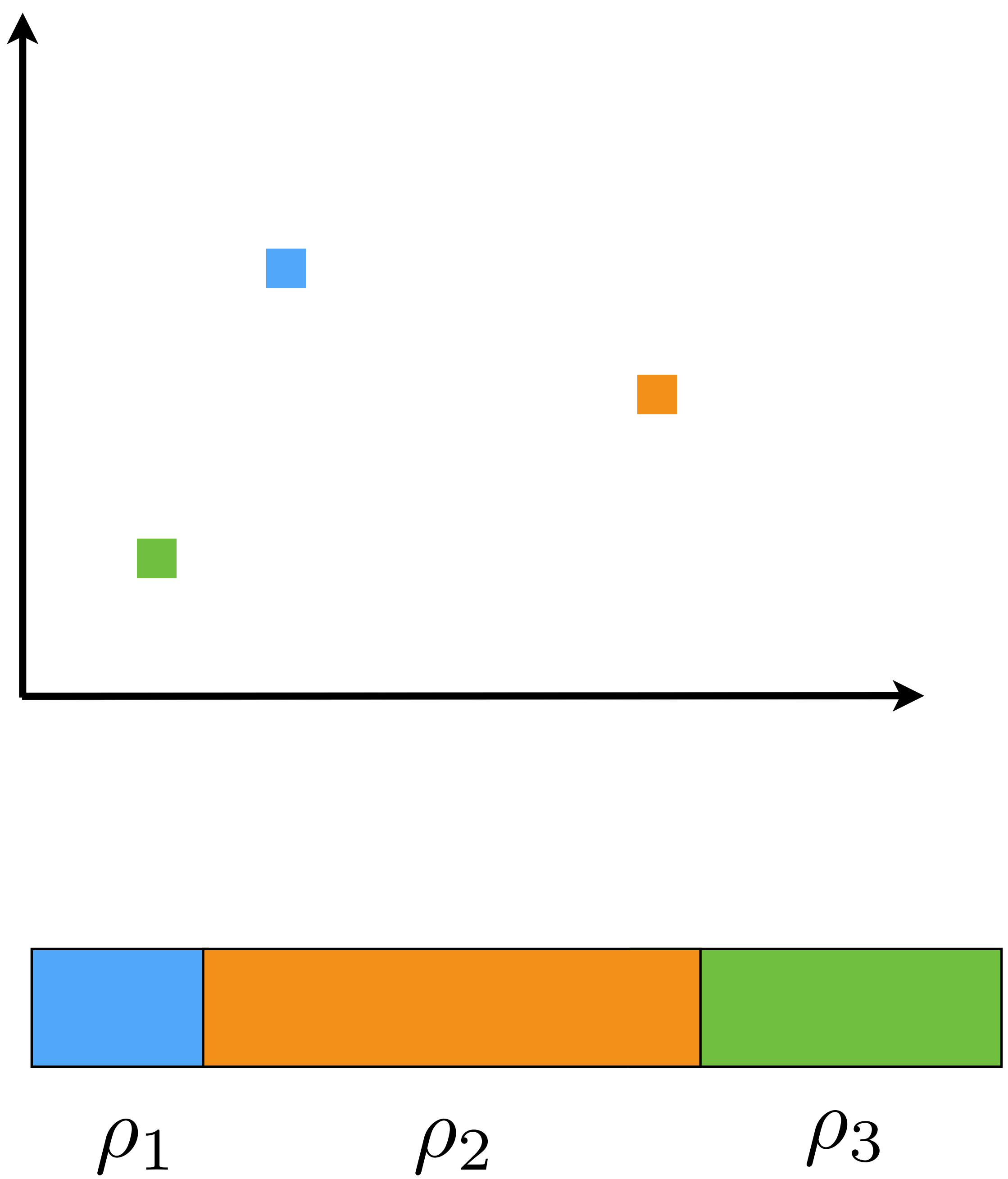

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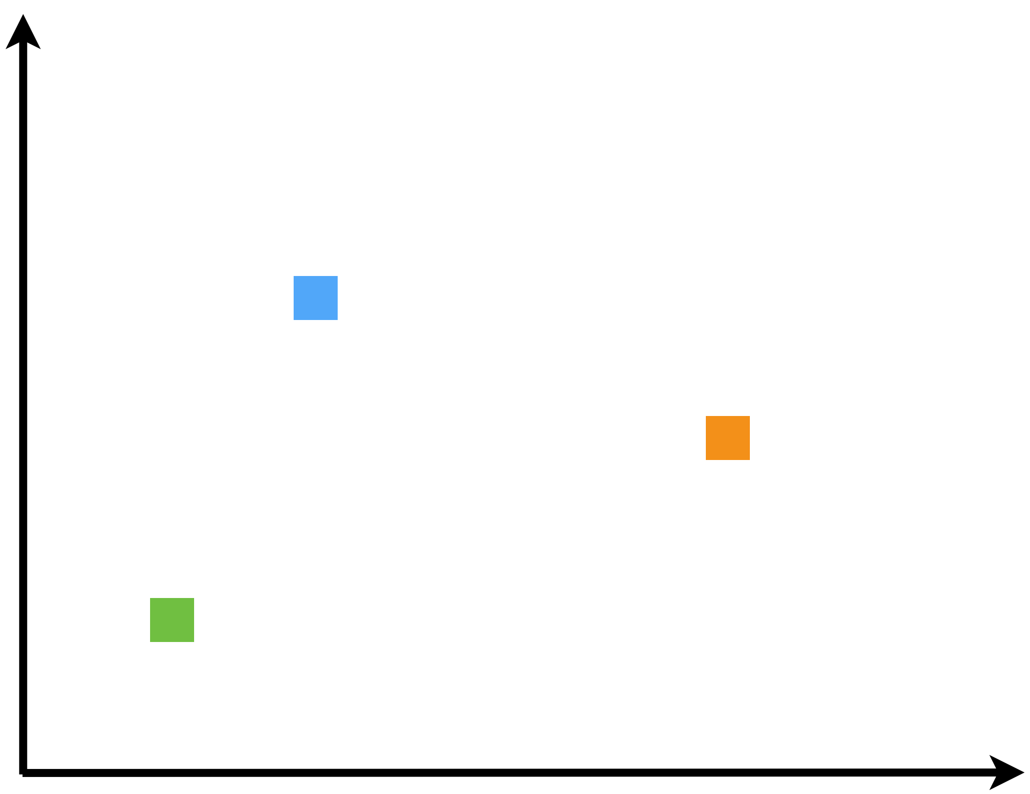

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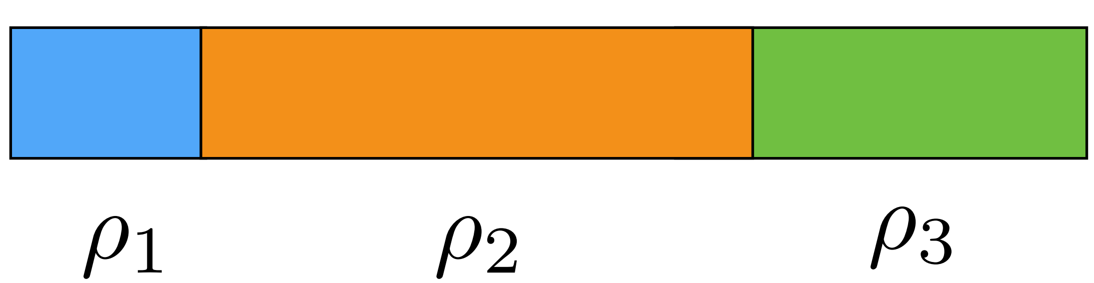

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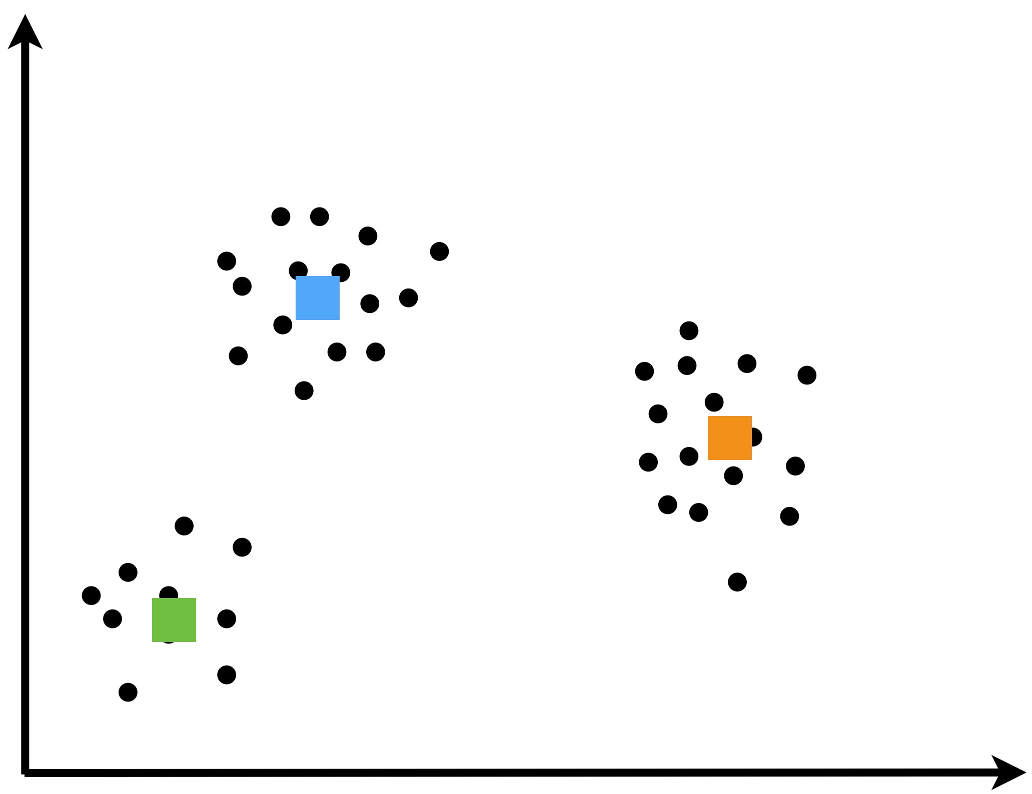

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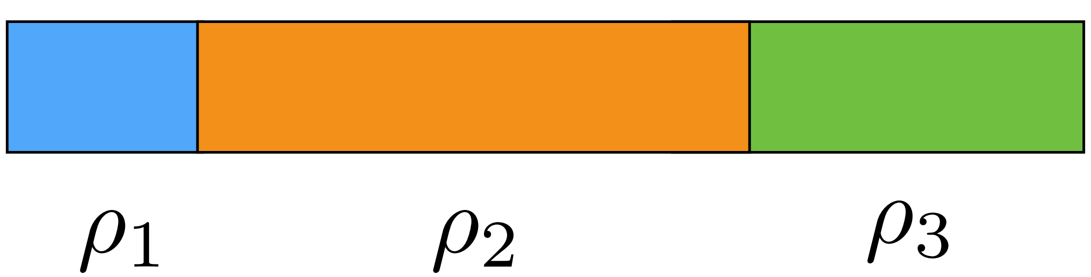

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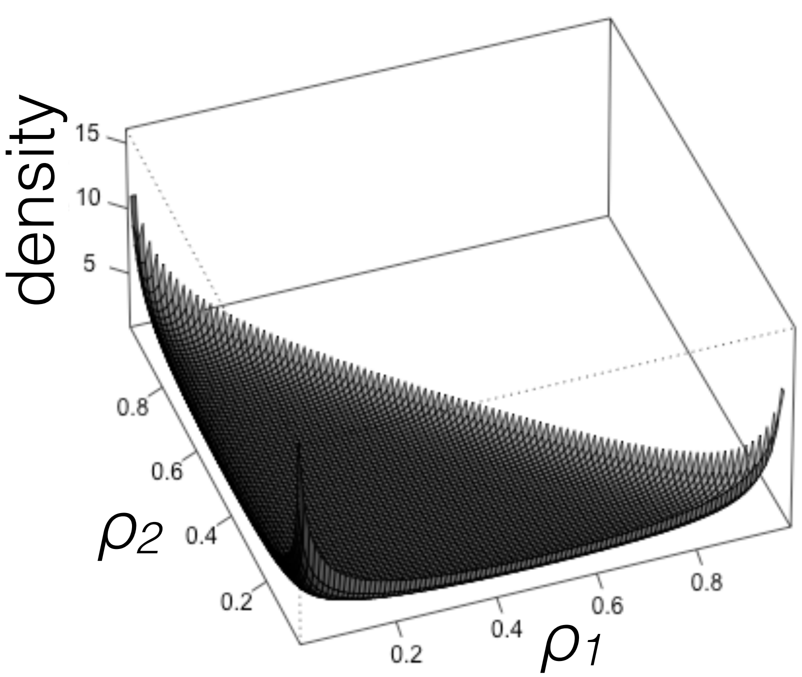
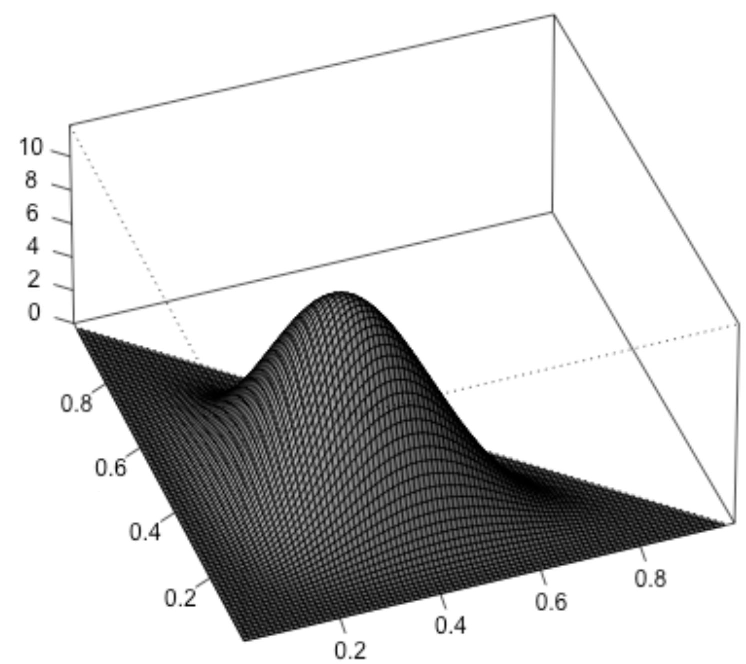
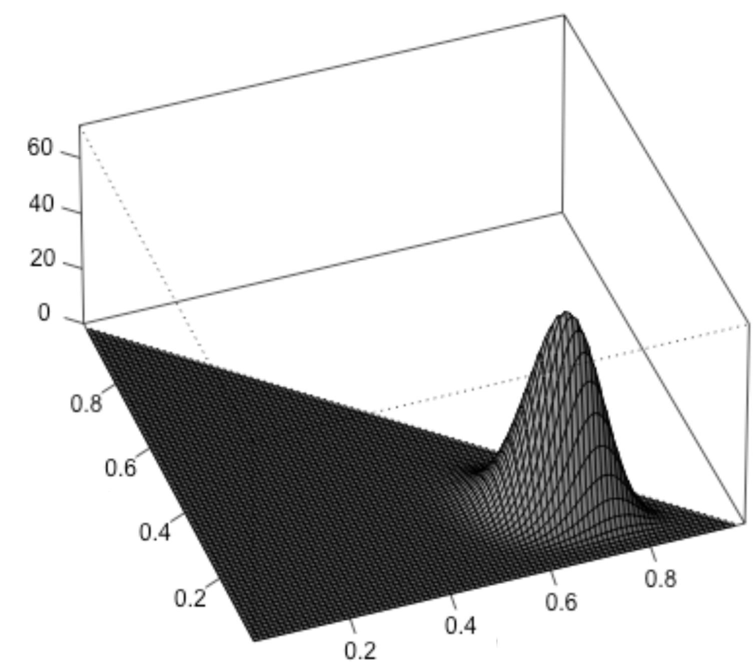


a = (5,5,5)



a = (40,10,10)



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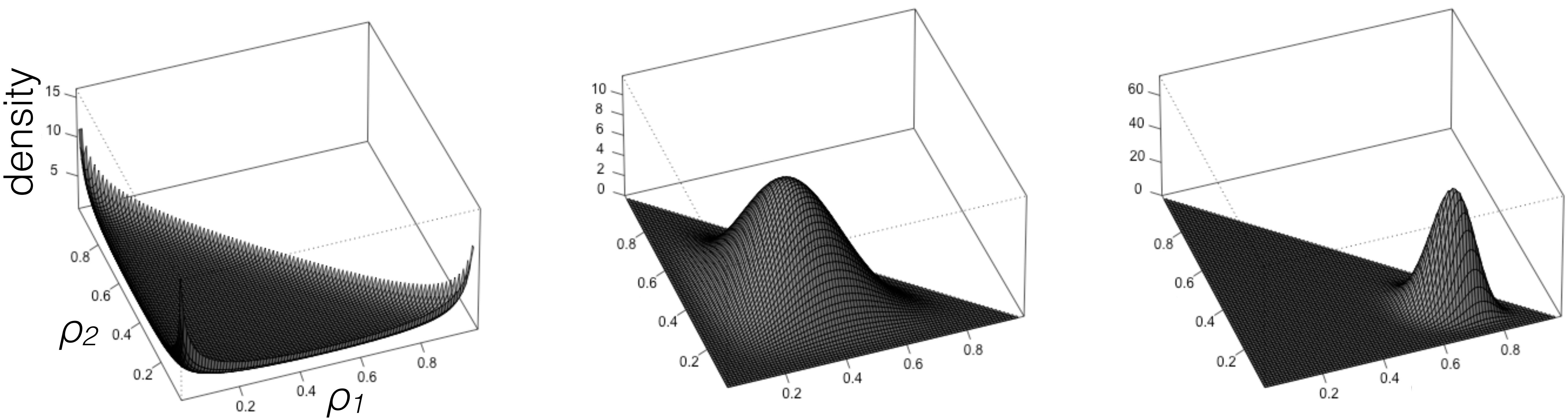

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[demo]

What if $K > N$?

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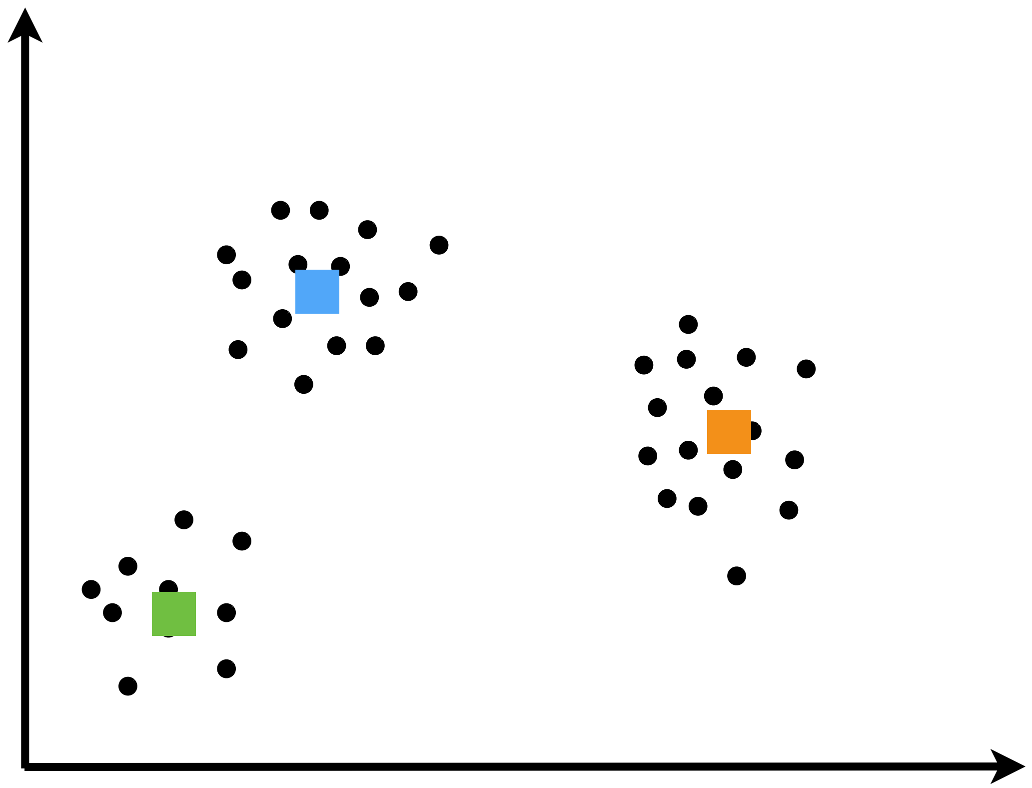

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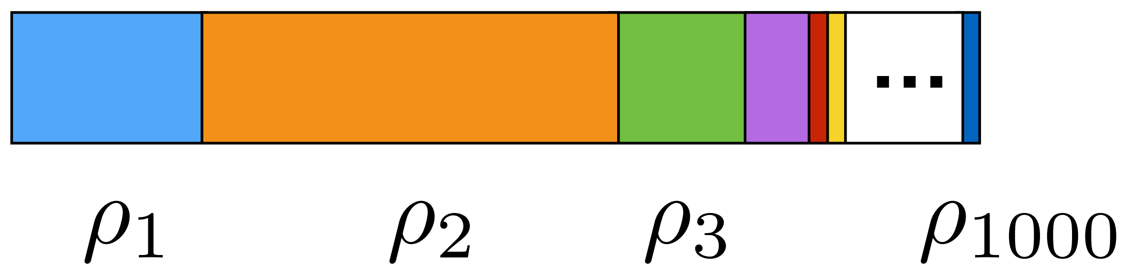

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- e.g. species sampling, topic modeling, groups on a social network, etc.

ρ_1 ρ_2 ρ_3 ρ_{1000}

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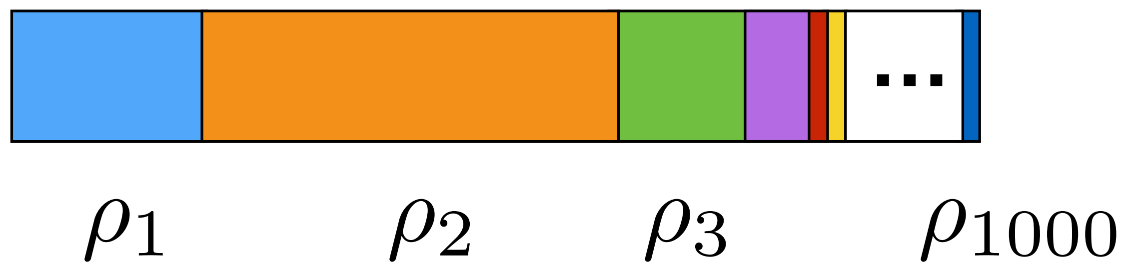

- Components: latent groups

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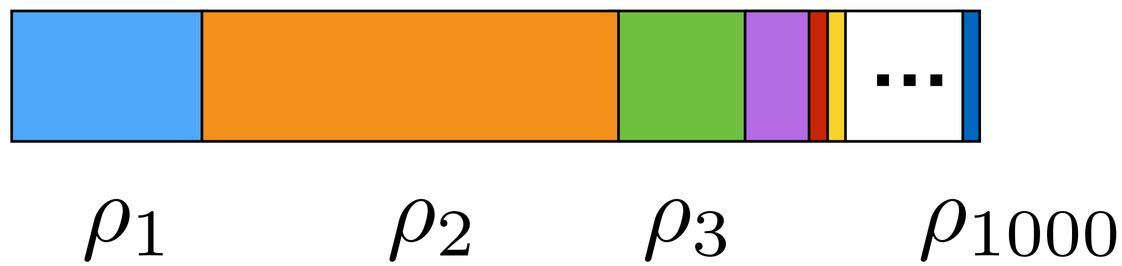

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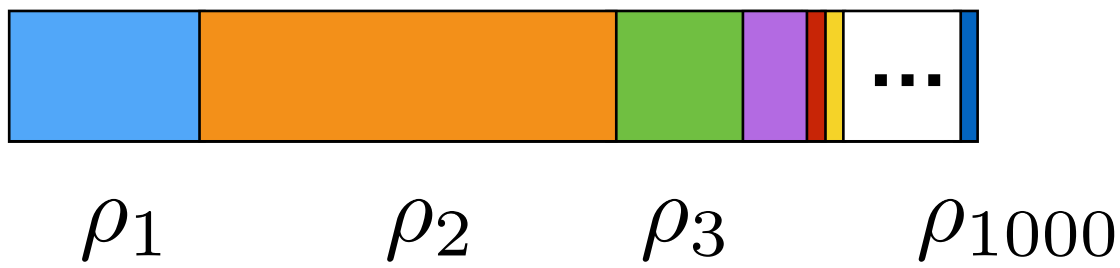

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- [demo 1, demo 2]

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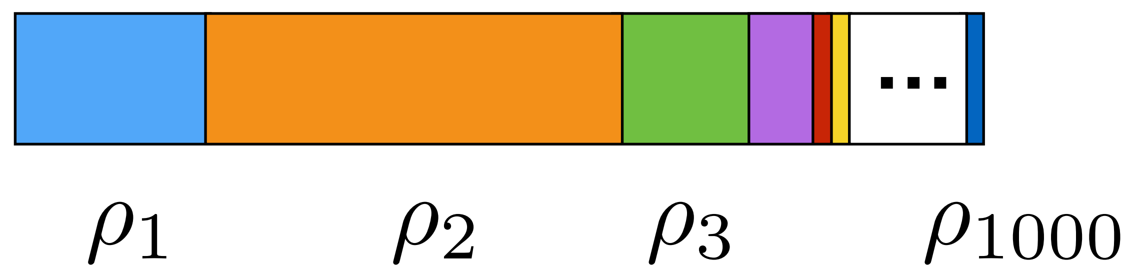

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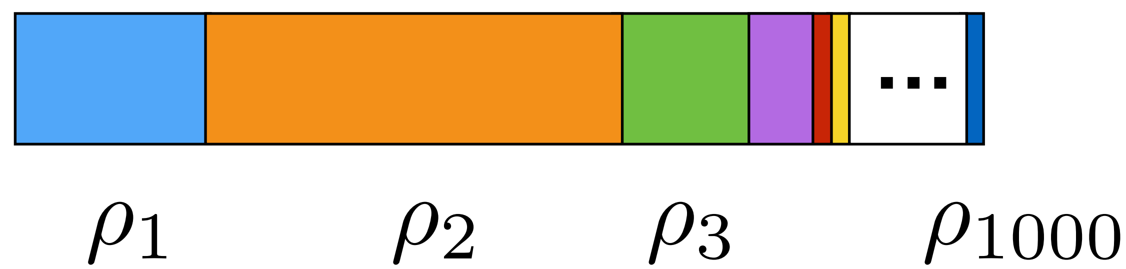

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- [demo 1, demo 2]
- Number of clusters for N data points is $< K$ and random
- Number of clusters grows with N

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data

Choosing $K = \infty$

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  - Observation: $\rho_{1:K} \sim \mathrm{Dirichlet}(a_{1:K})$

$$\Leftrightarrow \rho_1 \overset{d}{=} \mathrm{Beta}\left(a_1, \sum_{k=1}^{K} a_k - a_1\right) \perp\!\!\!\perp \frac{(\rho_2, \ldots, \rho_K)}{1-\rho_1} \overset{d}{=} \mathrm{Dirichlet}(a_2, \ldots, a_K)$$

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- "Stick breaking"

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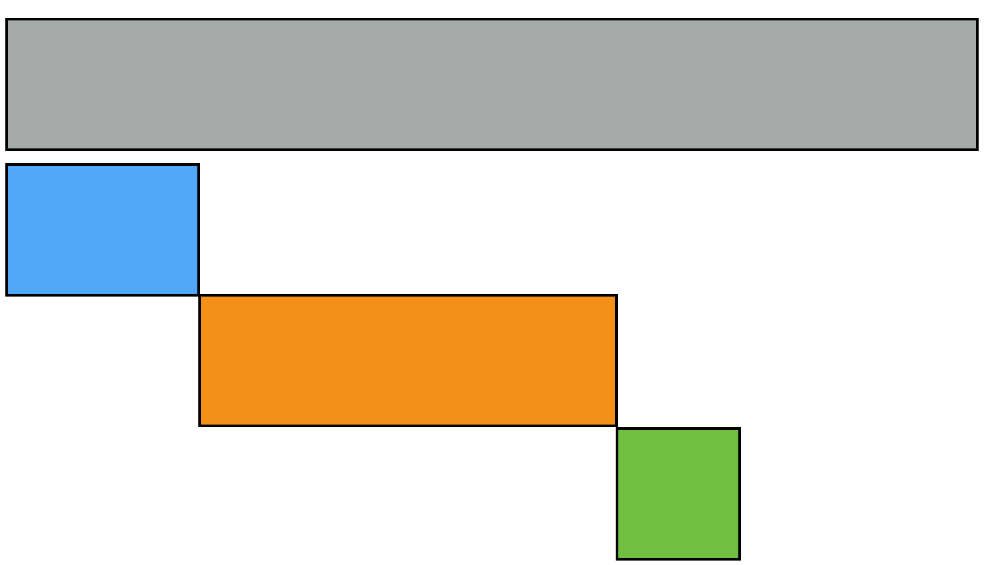

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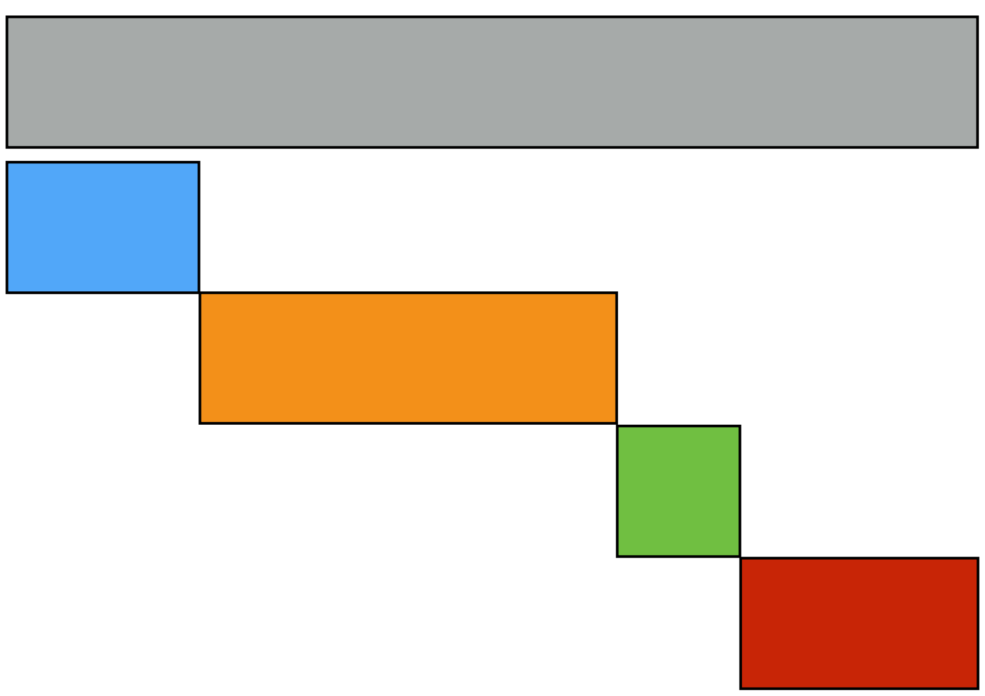

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$$\rho_4 = 1 - \sum_{k=1}^{3} \rho_k$$

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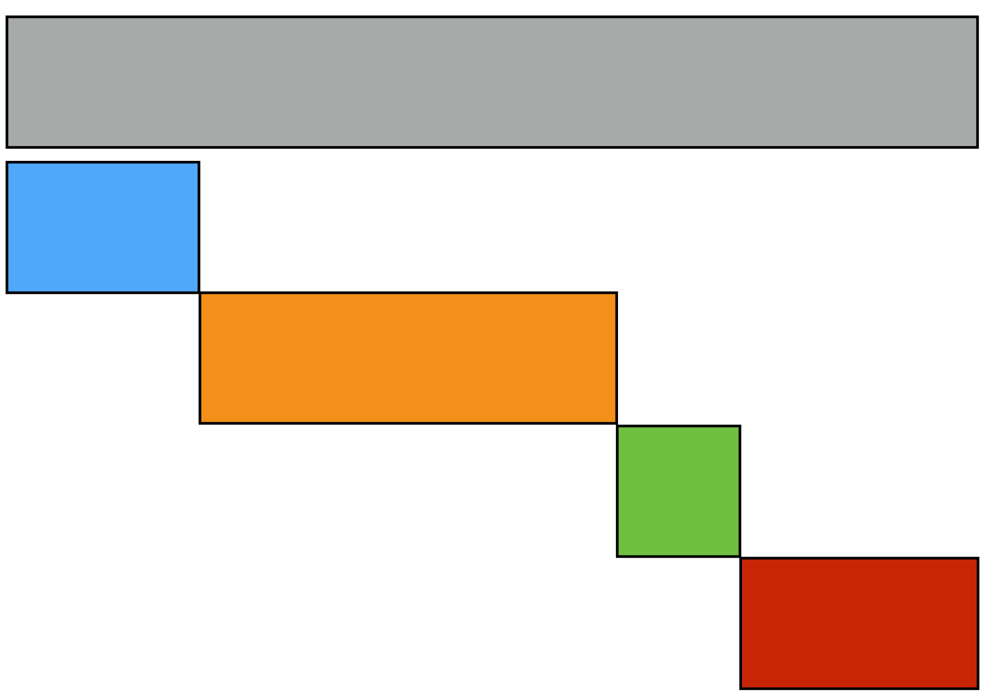

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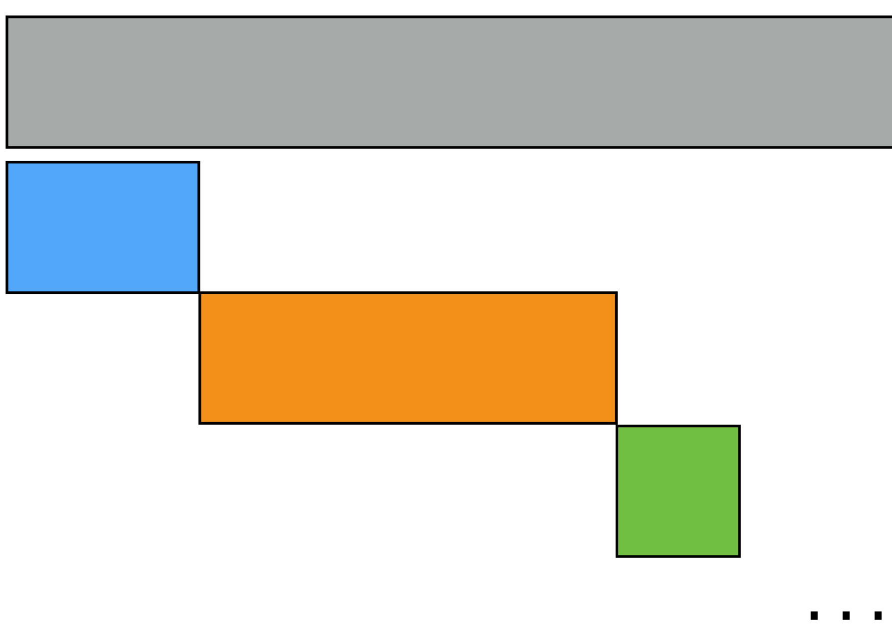

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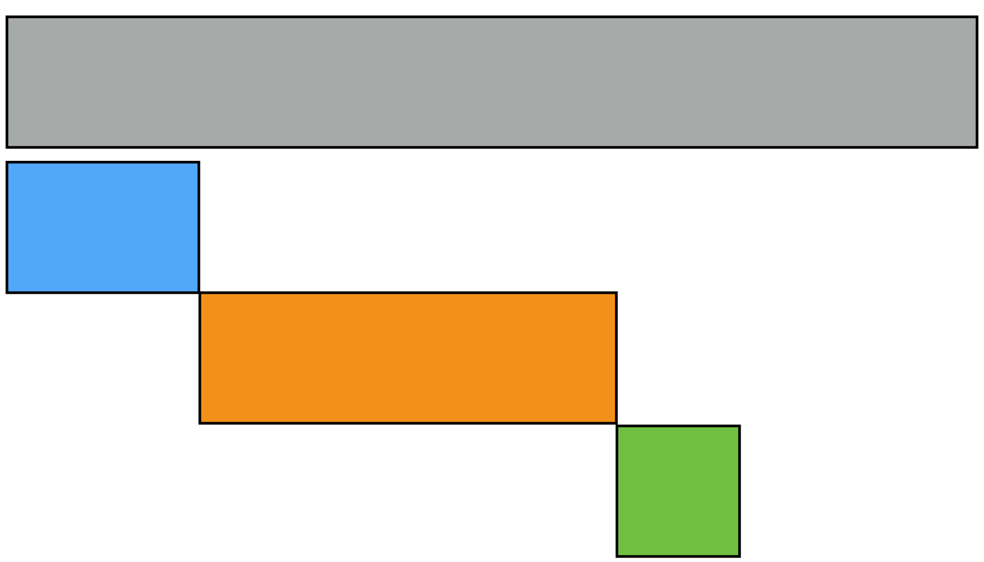

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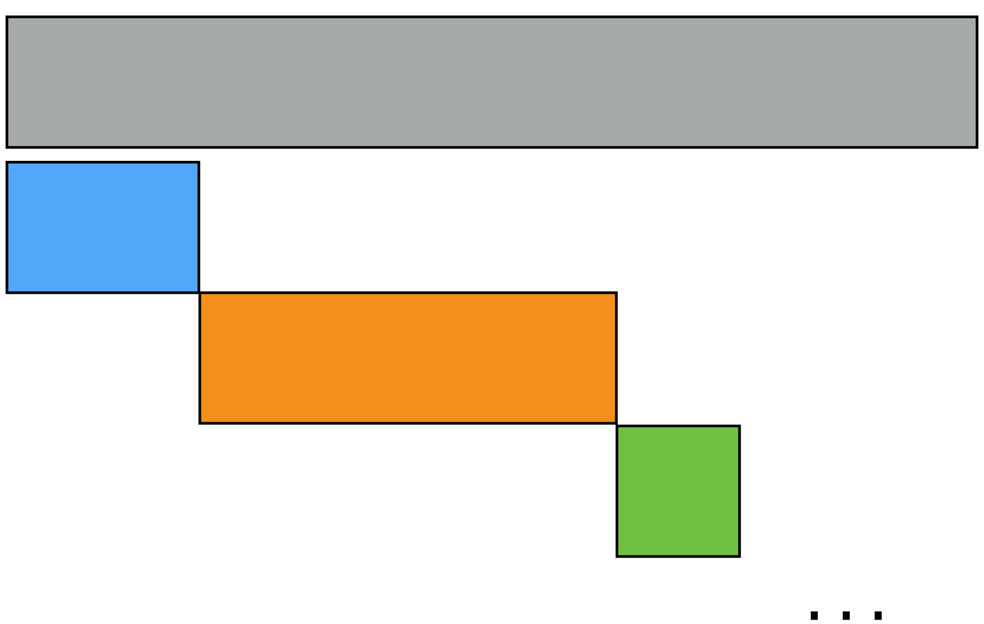

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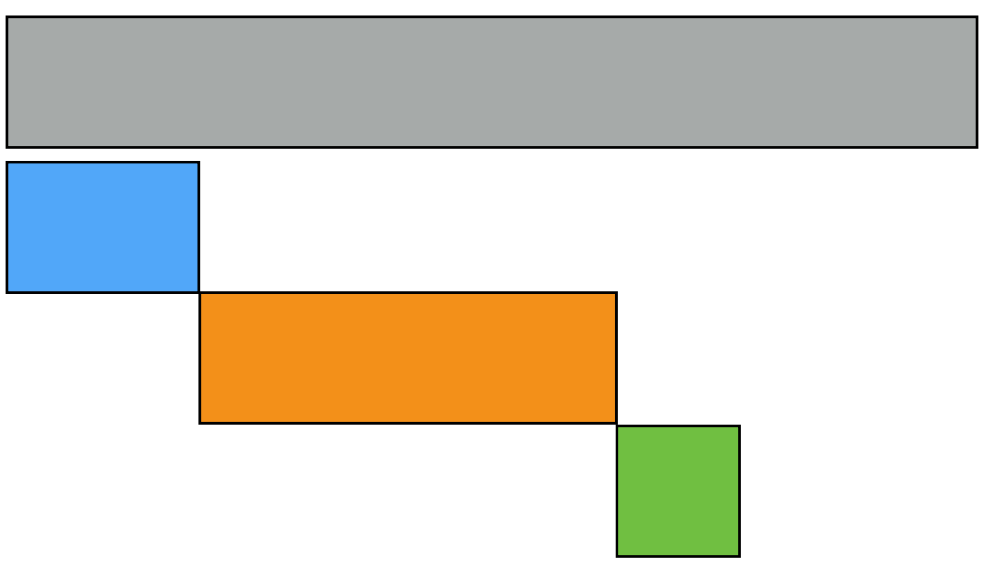

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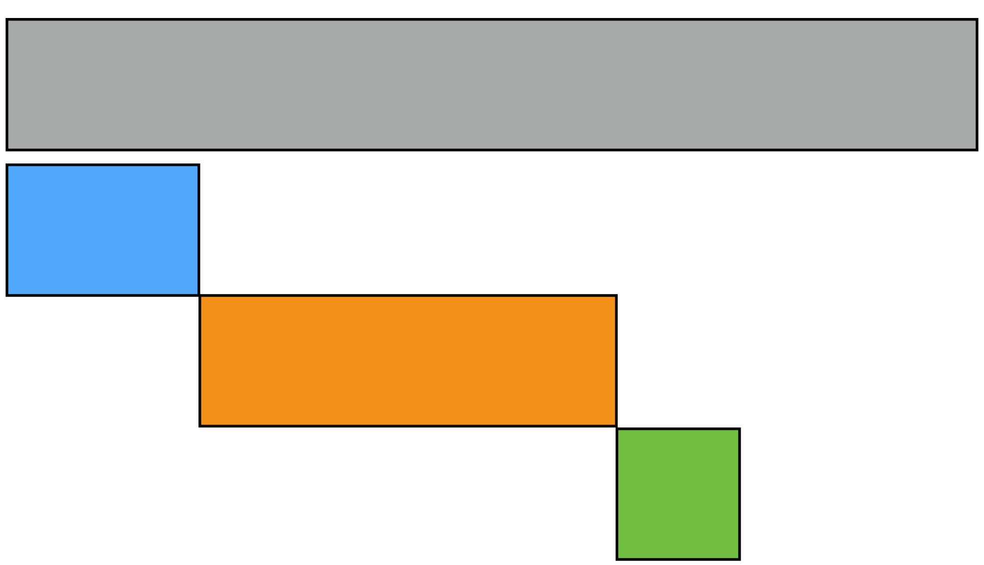

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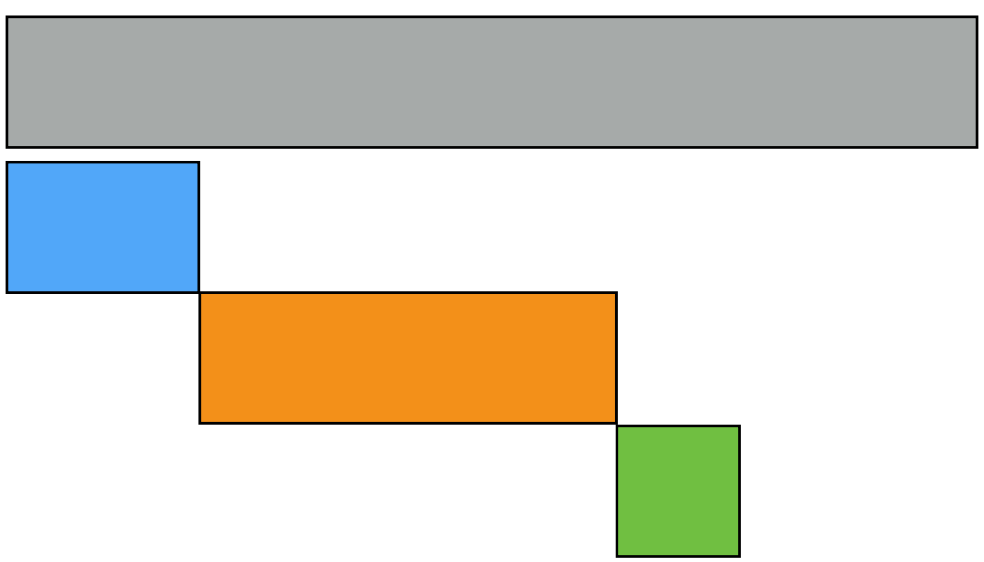

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[van der Vaart, Ghosal 2017]

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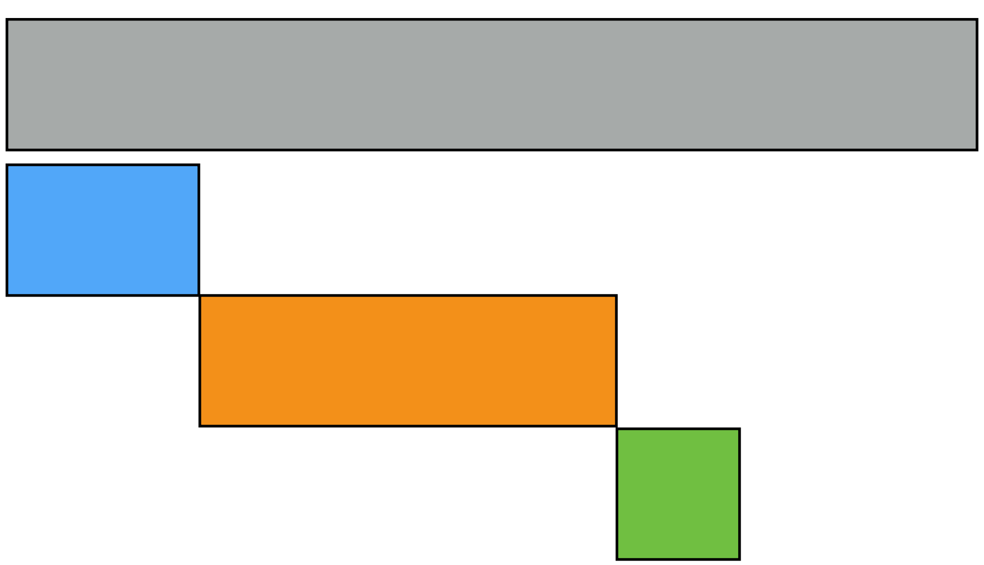

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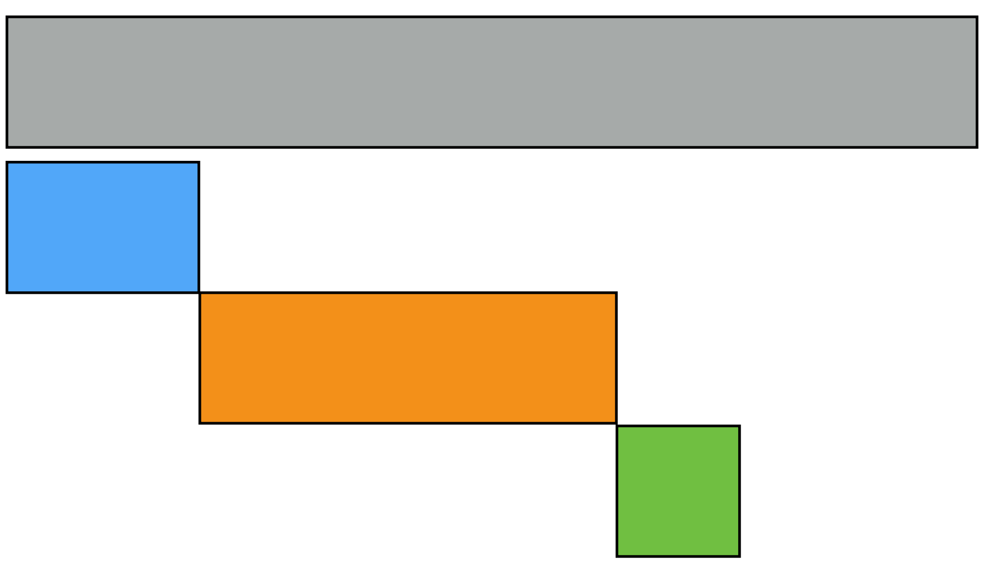

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[McCloskey 1965; Engen 1975; Patil and Taillie 1977; Ewens 1987; Sethuraman 1994; van der Vaart, Ghosal 2017]

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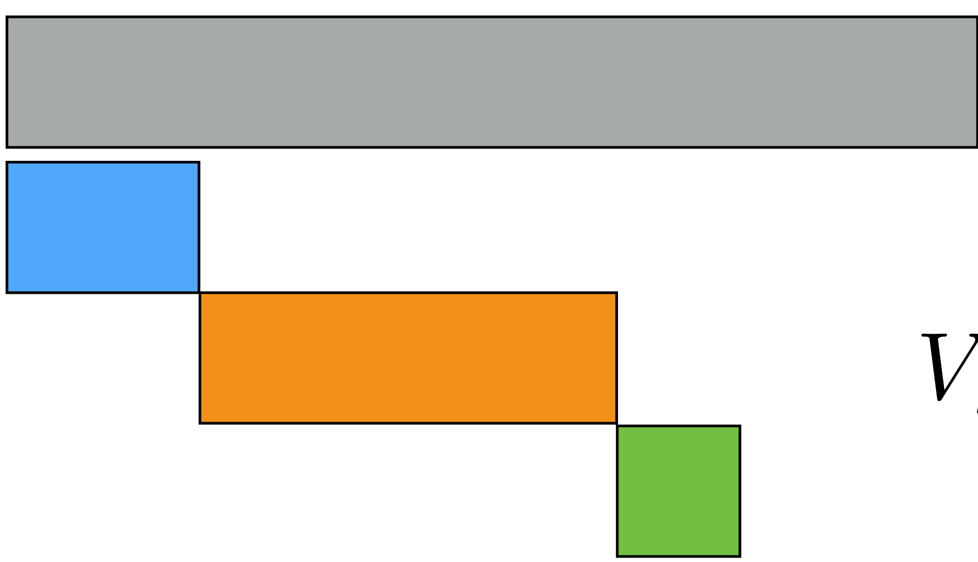

…

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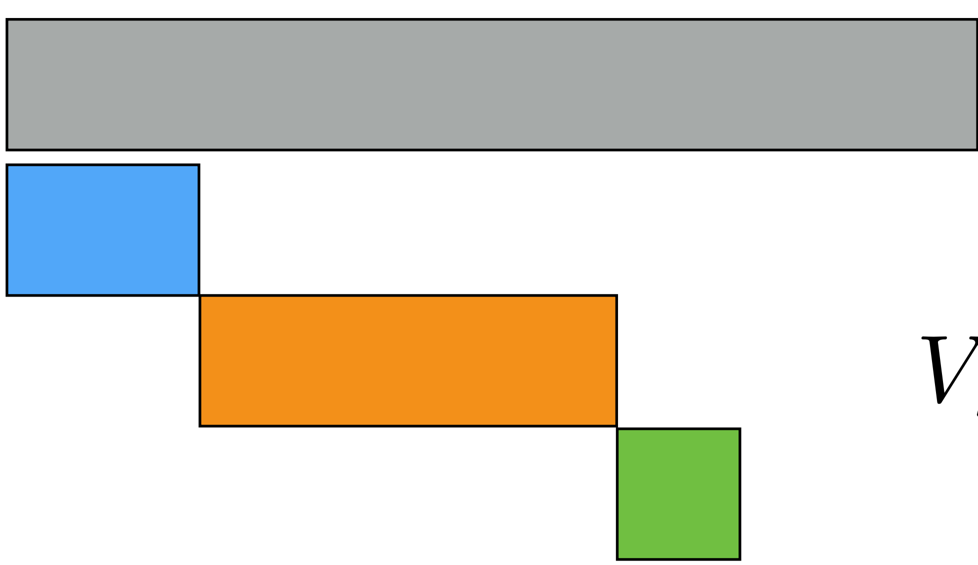

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[demo]

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References

- See Part II for full information